ETH Zürich
Chair of Software Engineering

Einführung in die Programmierung
Introduction to Programming

Prof. Dr. Bertrand Meyer

Exercise Session 10

News

- Mock exam in 2 weeks (December 6th, 7th)
  - You have to be present
  - The week after (last exercise session) we will discuss the results

Today

- Recursion
  - Recursion
    - Recursion
      - Recursion
        - Recursion
- Inheritance
- Genericity

Recursion: an example

- Fibonacci numbers:

  0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

- How can we calculate Fibonacci number *n*?

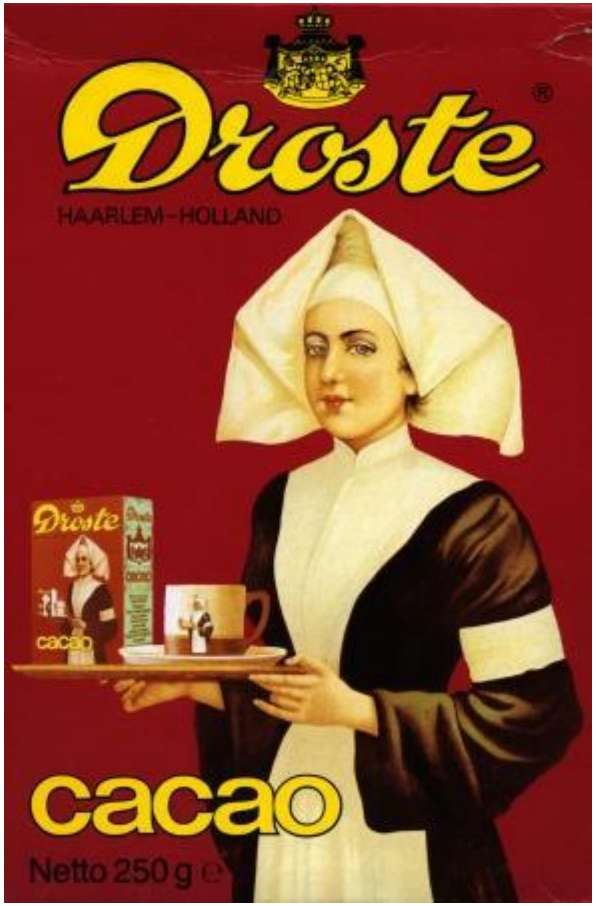
- Recursive formula:

$$F(n) = F(n-1) + F(n-2) \quad \text{for } n > 1$$

$$\text{with } F(0) = 0,\ F(1) = 1$$

Recursion: a second example

- Another example of recursion

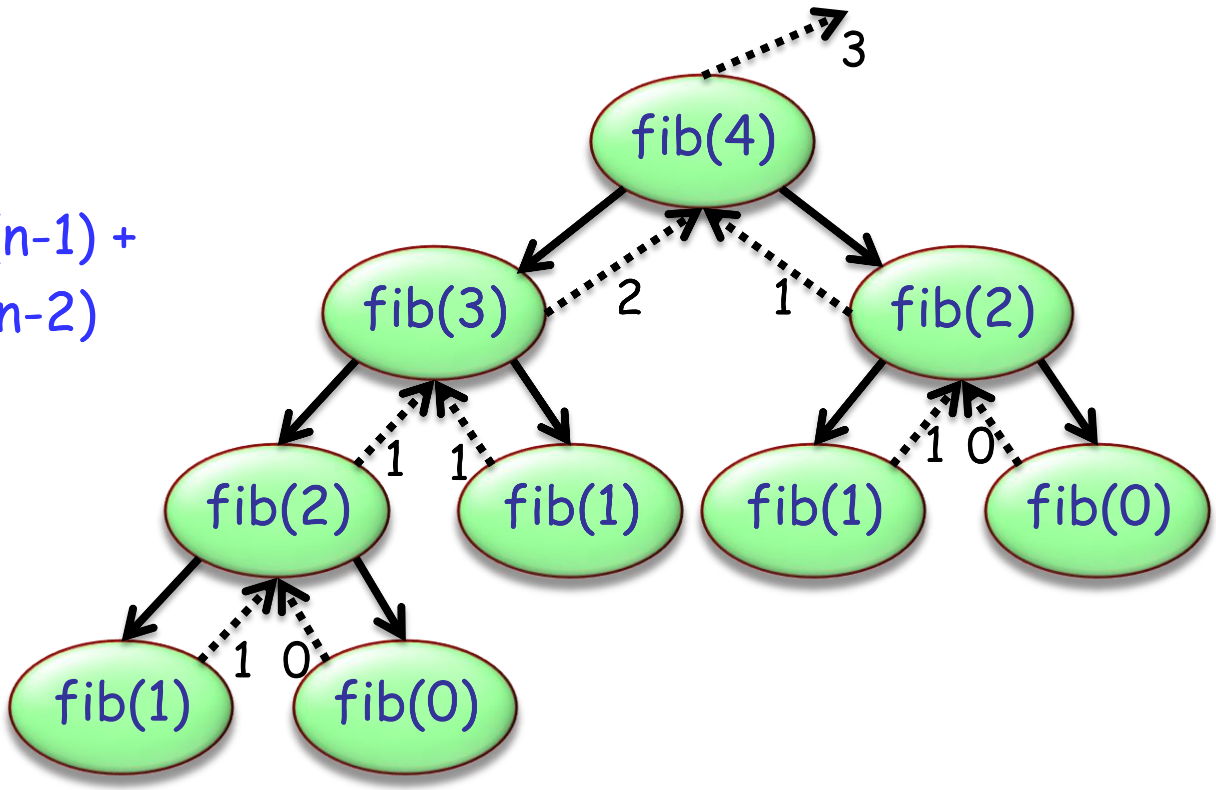


Source: en.wikipedia.org/wiki/Recursion

A recursive feature

```
fibonacci(n: INTEGER): INTEGER
  do
    if n = 0 then
      Result := 0
    elseif n = 1 then
      Result := 1
    else
      Result := fibonacci(n-1) +
                fibonacci(n-2)
  end
```

- Calculate fibonacci(4)



The general notion of recursion

A definition for a concept is recursive
if it involves an instance of the concept itself

- The definition may use more than one "*instance of the concept itself*"
- *Recursion* is the use of a recursive definition

Thoughts

„To iterate is human, to recurse - divine!“

but … computers are built by humans

Better use iterative approach if reasonable?

Iteration vs. recursion

- Every recursion could be rewritten as an iteration and vice versa.
- BUT, depending on how the problem is formulated, this can be difficult or might not give you a performance improvement.

Exercise: Printing numbers

Hands-On

- If we pass n = 4, how many numbers will be printed and in which order?

```
print_int (n: INTEGER)
    do
        print (n)
        if n > 1 then
            print_int (n - 1)
        end
    end
```

4321

```
print_int (n: INTEGER)
    do
        if n > 1 then
            print_int (n - 1)
        end
        print (n)
    end
```

1234

Exercise: Reverse string

Hands-On

- Print a given string in reverse order using a recursive function.

Exercise: Solution

```
class APPLICATION

create
    make

feature
    make
        local
            s: STRING
        do
            create s.make_from_string ("poldomangia")
            invert(s)
        end

    invert (s: STRING)
        require
            s /= Void
        do
            if not s.is_empty then
                invert (s.substring (2, s.count))
                print (s[1])
            end
        end
end
```

Exercise: Sequences

Hands-On

- Write a recursive and an iterative program to print the following:

  111,112,113,121,122,123,131,132,133,
  211,212,213,221,222,223,231,232,233,
  311,312,313,321,322,323,331,332,333,

- Note that the recursive solution can use loops too.

Exercise: Recursive solution

```
cells: ARRAY [INTEGER]

handle_cell (n: INTEGER)
    local
        i: INTEGER
    do
        from
            i := 1
        until
            i > 3
        loop
            cells [n] := i
            if (n < 3) then
                handle_cell (n+1)
            else
                print (cells [1].out+cells [2].out+cells [3].out+",")
            end
            i := i + 1
        end
    end
```

Exercise: Iterative solution

```
from
    i := 1
until
    i > 3
loop
    from
        j := 1
    until
        j > 3
    loop
        from
            k := 1
        until
            k > 3
        loop
            print (i.out+j.out+k.out+",")
            k := k + 1
        end
        j := j + 1
    end
    i := i + 1
end
```

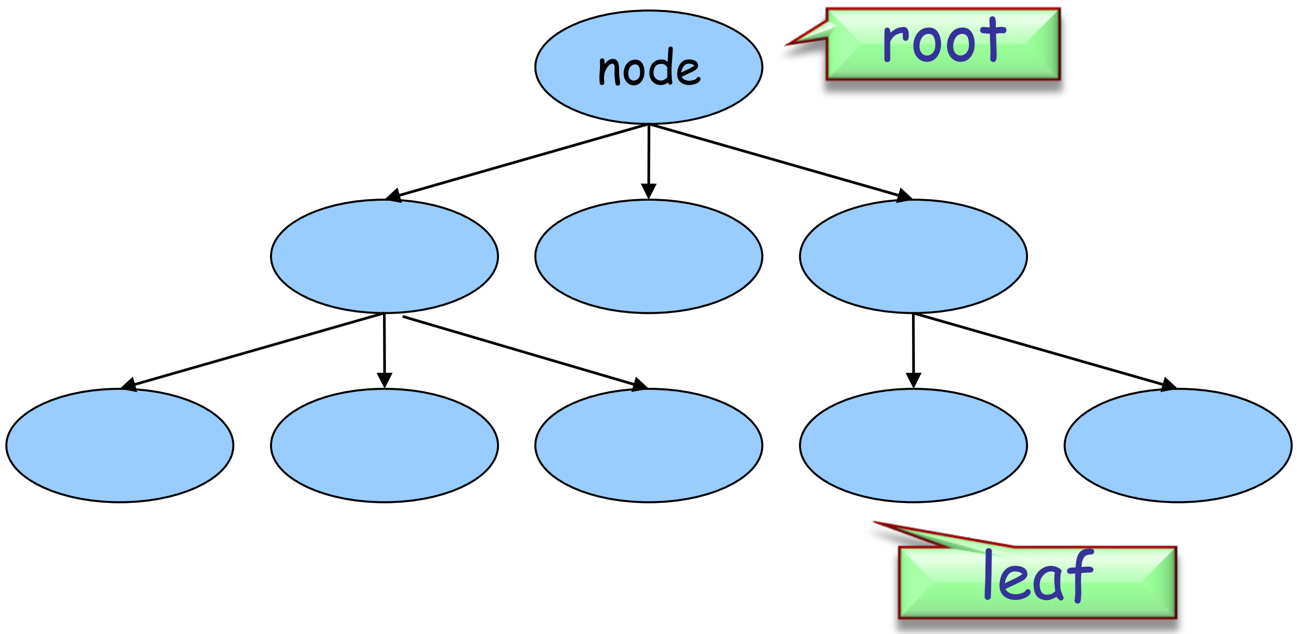
Data structures

- You have seen several data structures
  - *ARRAY*, *LINKED_LIST*, *HASH_TABLE*, …
- We will now look at another data structure and see how recursion can be used for traversal.

Tree

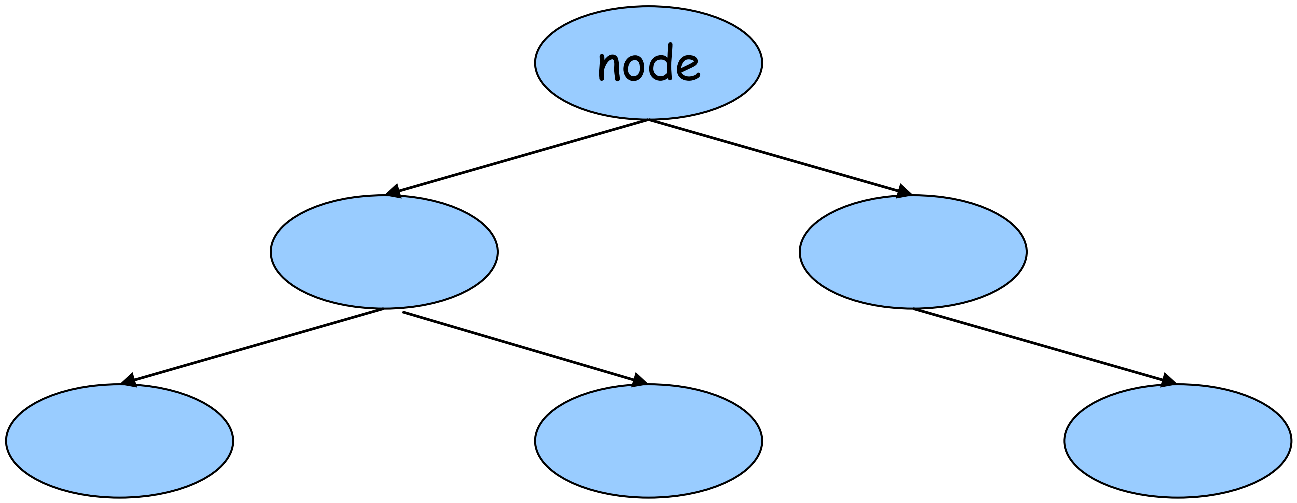
Tree

Tree: A more abstract way



- A non-empty tree has one root. An empty tree does not have a root.
- Every non-leaf node has links to its children. A leaf does not have children.
- There are no cycles.

Binary tree



- A binary tree is a tree.
- Each non-leaf node can have at most 2 children (possibly 0 or 1).

Exercise: Recursive traversal

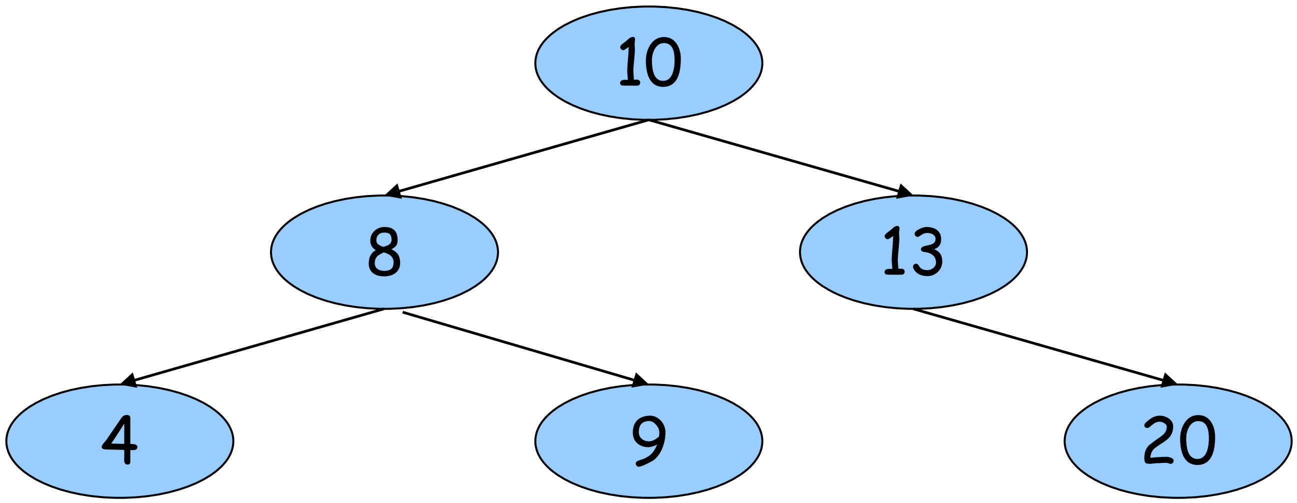
Hands-On

- Implement class *NODE* with an *INTEGER* attribute.
- In *NODE* implement a recursive feature that traverses the tree and prints out the *INTEGER* value of each *NODE* object.
- Test your code with a class *APPLICATION* which builds a binary tree and calls the traversal feature.

Exercise: Solution

- See code in IDE.

Binary search tree



- A binary search tree is a binary tree where each node has a *COMPARABLE* value.
- Left sub-tree of a node contains only values less than the node's value.
- Right sub-tree of a node contains only values greater than or equal to the node's value.

Exercise: Adding nodes

Hands-On

- Implement command *put (n: INTEGER)* in class *NODE* which creates a new *NODE* object at the correct place in the binary search tree rooted by **Current**.
- Test your code with a class *APPLICATION* which builds a binary search tree using put and prints out the values using the traversal feature.
- Hint: You might need to adapt the traversal feature such that the values are printed out in order.

Exercise: Solution

- See code in IDE.

Exercise: Searching

Hands-On

- Implement feature *has (n: INTEGER): BOOLEAN* in class *NODE* which returns true if and only if *n* is in the tree rooted by **Current**.
- Test your code with a class *APPLICATION* which builds a binary search tree and calls *has*.

Exercise: Solution

- See code in IDE.

Today

- Recursion
  - Recursion
    - Recursion
      - Recursion
        - Recursion
- Inheritance
- Genericity

Inheritance

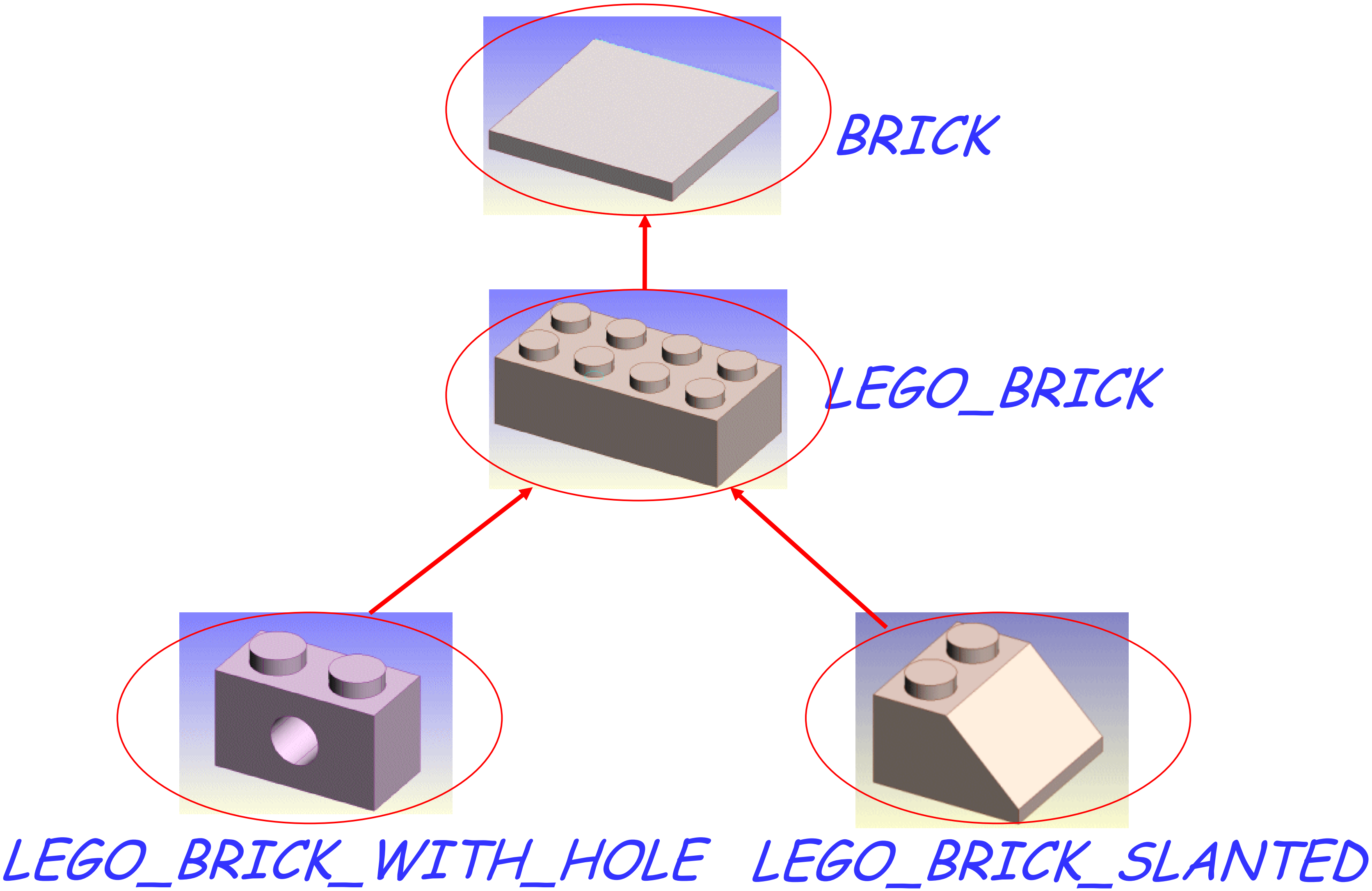
Principle:

Describe a new class as extension or specialization of an existing class
(or several with ***multiple*** inheritance)

If *B* inherits from *A* :

- As modules: all the services of *A* are available in *B* (possibly with a different implementation)
- As types: whenever an instance of *A* is required, an instance of *B* will be acceptable
("is-a" relationship)

Let's play Lego!

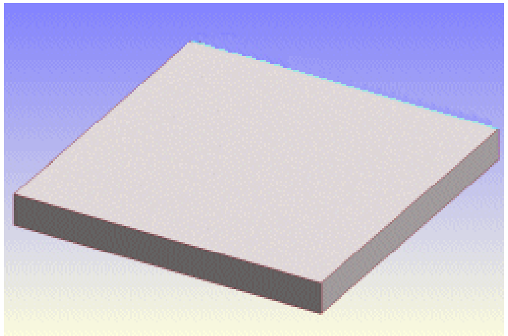


Class *BRICK*

```
deferred class
    BRICK

feature
    width: INTEGER
    depth: INTEGER
    height: INTEGER
    color: COLOR

    volume: INTEGER
        deferred
        end
end
```



Class *LEGO_BRICK*

Inherit all features of class *BRICK*.

New feature, calculate all nubs

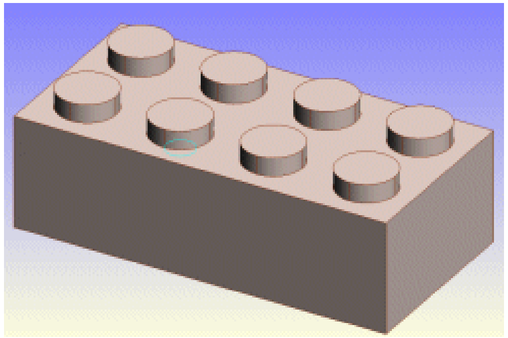
Implementation of *volume*.

```
class
    LEGO_BRICK

inherit
    BRICK

feature
    number_of_nubs: INTEGER
        do
            Result := ...
        end

    volume: INTEGER
        do
            Result := ...
        end
end
```



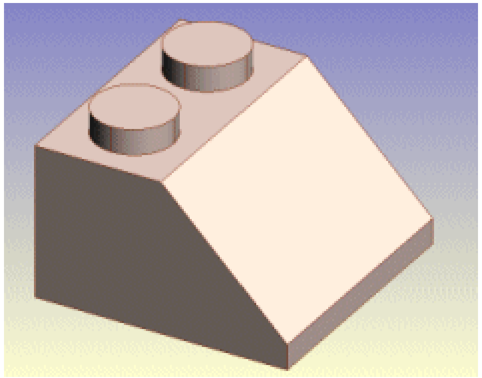
Class *LEGO_BRICK_SLANTED*

```
class
    LEGO_BRICK_SLANTED

inherit
    LEGO_BRICK
        redefine
            volume
        end

feature
    volume: INTEGER
        do
            Result := ...
        end
end
```

The feature *volume* is going to be redefined (=changed). The feature *volume* comes from *LEGO_BRICK*



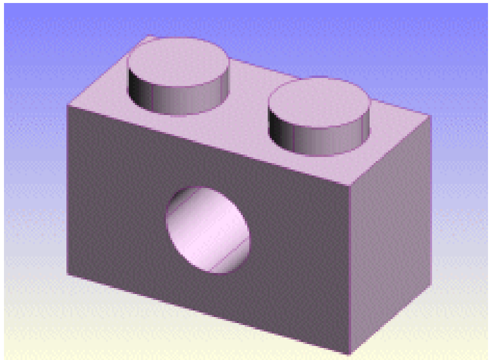
Class *LEGO_BRICK_WITH_HOLE*

```
class
    LEGO_BRICK_WITH_HOLE

inherit
    LEGO_BRICK
        redefine
            volume
        end

feature
    volume: INTEGER
        do
            Result := ...
        end
end
```

The feature *volume* is going to be redefined (=changed). The feature *volume* comes from *LEGO_BRICK*



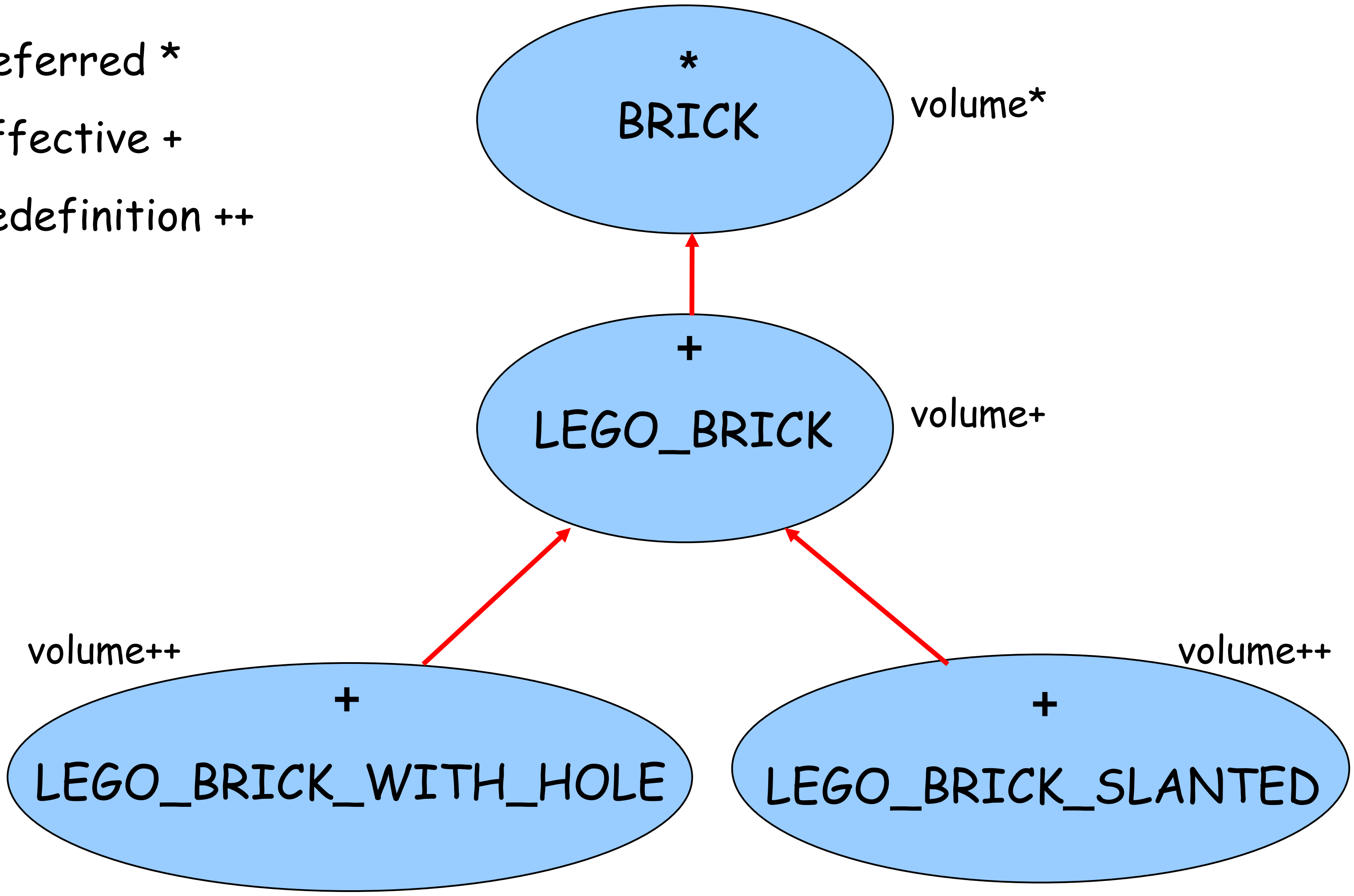
Inheritance Notation

Notation:

Deferred *

Effective +

Redefinition ++



Deferred

- Deferred
  - Deferred classes can have deferred features.
  - A class with at least one deferred feature must be declared as deferred.
  - A deferred feature does not have an implementation yet.
  - Deferred classes cannot be instantiated and hence cannot contain a create clause.

Effective

- Effective
  - Effective classes do not have deferred features (the “standard case”).
  - Effective routines have an implementation of their feature body.

Precursor

- If a feature was redefined, but you still wish to call the old one, use the **Precursor** keyword.

```
volume: INTEGER
  do
     Result := Precursor - ...
  end
```

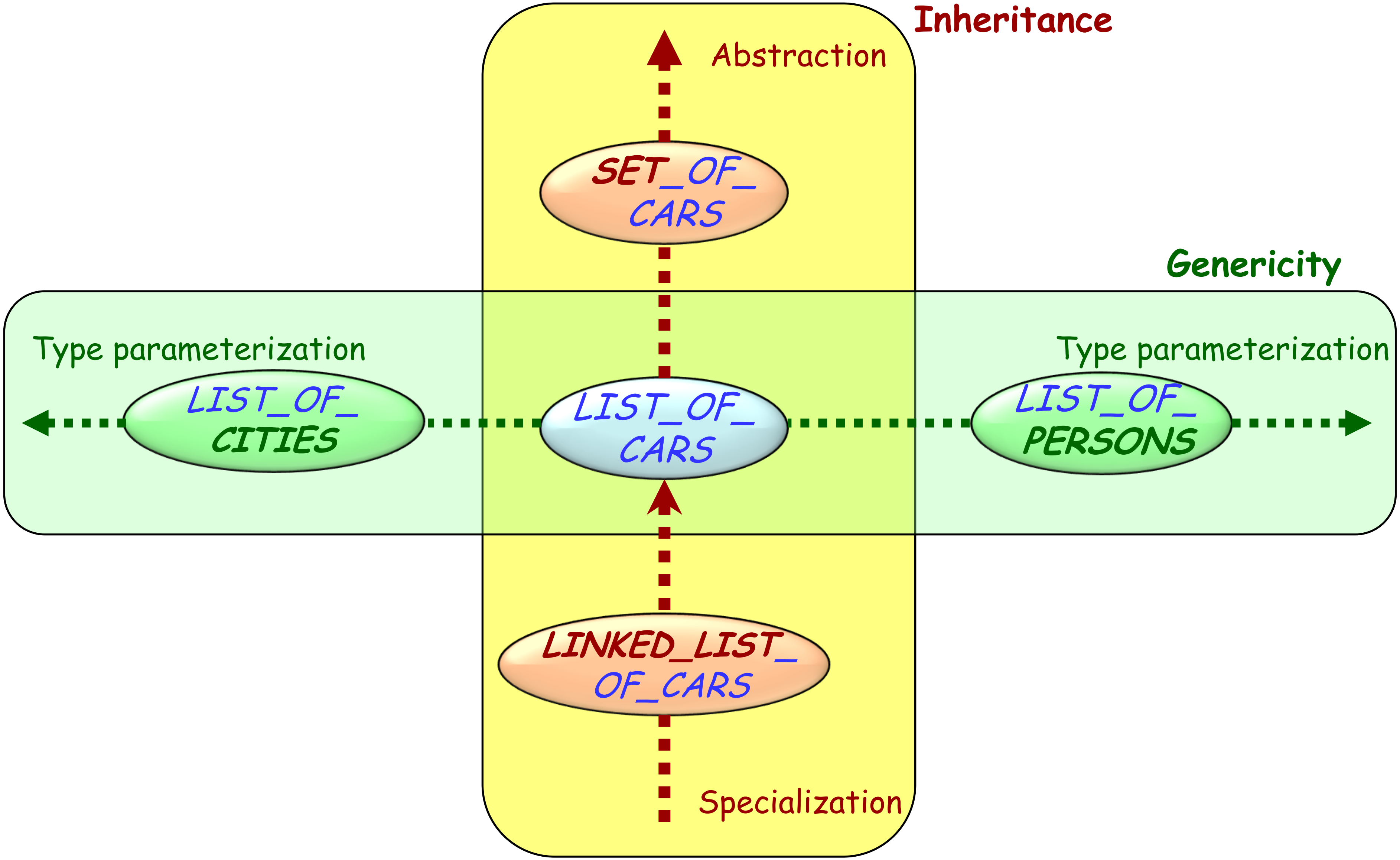
Today

- Recursion
  - Recursion
    - Recursion
      - Recursion
        - Recursion
- Inheritance
- Genericity

Genericity

- Genericity lets you parameterize a class. The parameters are types. A single class text may be reused for many different types.

Genericity



Inheritance
Abstraction
SET_OF_
CARS
Genericity
Type parameterization
Type parameterization
LIST_OF_
CITIES
LIST_OF_
CARS
LIST_OF_
PERSONS
LINKED_LIST_
OF_CARS
Specialization

A generic list

Formal generic parameter

class *LIST* [*G*] **feature**

extend (*x* : *G*) ...

last : *G* ...

end

To use the class: obtain a generic derivation, e.g.

Actual generic parameter

cities : *LIST* [*CITY*]

A generic list with constraints

```
class
    STORAGE [G -> RESOURCE]

inherit
    LIST [G]

feature
    consume_all
        do
            from start until after
            loop
                item.consume
                forth
            end
        end
end
```

constrained generic parameter

The feature *item* is of type G. We cannot assume *consume*.

assume this.

Type-safe containers

- Using genericity you can provide an implementation of type safe containers. This helps avoiding object-tests.

```
x: ANIMAL
animal_list: LINKED_LIST [ANIMAL]
a_rock: MINERAL

animal_list.put (a_rock) -- Does this rock?
```

The End

End of slides

Time left?

Here's another recursion examples...

Exercise: Magic Squares

- A magic square of size NxN is a NxN square such that:
  - Every cell contains a number between 1 and N^2.
  - The sum in every row and column is constant.
  - The numbers are all different.

4	**3**	**8**
9	5	1
2	7	6

Exercise: Magic Squares

- Finding a 3x3 magic square is related to finding the permutations of 1 to 9.
- There exist 72 magic 3x3 squares.

123456789
123456798
123456879
123456897
123456978
123456987
...
987654321

Exercise: Magic Squares

Hands-On

- Write a program that finds all the 3x3 magic squares.

- Hints
  - Reuse the previous recursive algorithm by applying it to permutations (enforce no repetitions).
  - Use two arrays of 9 elements, one for the current permutation and one to know if a number has already been used or not.

Exercise: Solution

- See code in IDE.