Software Verification
Exercise Solution: Separation Logic

By application of small axioms and the frame rule, we can obtain the following proof outline:

copytree(i; j) =
{tree τ i}
if i = nil then
{tree τ i ∧ i = nil}
{τ = ε ∧ empty ∧ i = nil}
{τ = ε ∧ (empty ∧ i = nil) * (empty ∧ i = nil)}
j := i
{τ = ε ∧ (empty ∧ i = nil) * (empty ∧ j = nil)}
{τ = ε ∧ (tree ε i) * (tree ε j)}
{tree τ i * tree τ j}
else
newvar i_1, i_2, v, j_1, j_2 in
{tree τ i ∧ i != nil}
{∃j,a,k,τ_1,τ_2. (i↦j,a,k) * (tree τ_1 j) * (tree τ_2 k) ∧ τ = (τ_1,a,τ_2)}
i_1 := [i];
{∃j,a,k,τ_1,τ_2. (i↦j,a,k) * (tree τ_1 j) * (tree τ_2 k) ∧ τ = (τ_1,a,τ_2) ∧ i_1 = j}
{∃a,k,τ_1,τ_2. (i↦i_1,a,k) * (tree τ_1 i_1) * (tree τ_2 k) ∧ τ = (τ_1,a,τ_2)}
v := [i + 1];
{∃k,τ_1,τ_2. (i↦i_1,v,k) * (tree τ_1 i_1) * (tree τ_2 k) ∧ τ = (τ_1,v,τ_2)}
i_2 := [i + 2];
{∃τ_1,τ_2. (i↦i_1,v,i_2) * (tree τ_1 i_1) * (tree τ_2 i_2) ∧ τ = (τ_1,v,τ_2)}
copytree(i_1, j_1);
{∃τ_1,τ_2. (i↦i_1,v,i_2) * (tree τ_1 i_1) * (tree τ_1 j_1) * (tree τ_2 i_2) ∧ τ = (τ_1,v,τ_2)}
copytree(i_2, j_2);
{∃τ_1,τ_2. (i↦i_1,v,i_2) * (tree τ_1 i_1) * (tree τ_1 j_1) * (tree τ_2 i_2) * (tree τ_2 j_2) ∧ τ = (τ_1,v,τ_2)}
j := cons(j_1, v, j_2);
{∃τ_1,τ_2. (i↦i_1,v,i_2) * (tree τ_1 i_1) * (tree τ_1 j_1) * (tree τ_2 i_2) * (tree τ_2 j_2) * (j↦j_1,v,j_2) ∧ τ = (τ_1,v,τ_2)}
{∃τ_1,τ_2. (i↦i_1,v,i_2) * (tree τ_1 i_1) * (tree τ_2 i_2) * (j↦j_1,v,j_2) * (tree τ_1 j_1) * (tree τ_2 j_2) ∧ τ = (τ_1,v,τ_2)}
{tree τ i * tree τ j}
end
end
{tree τ i * tree τ j}