Software Verification

Lecture 11: Verification of Real-time Systems

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Program Verification: the very idea

P: a program

```
max (a, b: INTEGER): INTEGER is
    do
        if a > b then
            Result := a
        else
            Result := b
        end
    end
```

S: a specification

```
require
    true

ensure
    Result >= a
    Result >= b
```

Does $P \models S$ hold?

The Program Verification problem:

- Given: a program P and a specification S
- Determine: if every execution of P, for every value of input parameters, satisfies S

Real-time Verification

P: a program

```
max (a, b: INTEGER): INTEGER is
    do
        if a > b then
            Result := a
        else
            Result := b
        end
    end
```

S: a specification

```
ensure
    Result >= a
    Result >= b

ensure -- real-time
"max terminates no sooner
 than 3 ms and no later than
 10 ms after invocation"
```

Does $P \models S$ hold?

The Real-time Verification problem:

- Given: program P (embedded in system E) and real-time specification S
- Determine: if every execution of P (within E) satisfies S

Real-time Programs and Systems

Def. Real-time specification: specification that includes exact timing information.

Def. Real-time computation: computation whose specification is real-time. In other words: computation whose correctness depends not only on the value of the result but also on when the result is available.

- The timing of a piece of software is usually dependent on the environment where the computation takes place
- Hence, in real-time verification the focus shifts from programs to (software-intensive) systems
  - In a system, even the physical environment is often relevant
- The purely computational aspects can often be analyzed in isolation
- Real-time verification can then focus on real-time aspects of the system
  - e.g., synchronization, deadlines, delays, ...

while abstracting away most of the rest

Decidability vs. Expressiveness Trade-Off

The Real-time Verification problem:

- Given: program P (embedded in system E) and real-time specification S
- Determine: if every execution of P (within E) satisfies S

P: a system

⇕

F(P): some formal model of P

S: a real-time specification

⇕

N(S): some formal notation for S

Does $F(P) \models N(S)$ hold?

- The classes for F(P) and N(S) should guarantee:
  - enough expressiveness to include a quantitative notion of time
  - decidability of the verification problem

Real-time Model-Checking

The Real-time Model Checking problem:

- Given: a timed automaton A and a metric temporal-logic formula F
- Determine: if every run of A satisfies F or not
  - if not, also provide a counterexample: a run of A where F does not hold

A: a timed automaton

F: a metric temporal-logic formula

$$A \overset{?}{\models} F$$

- The model-checking paradigm is naturally extended to real-time systems
- Different choices are possible for the family of automata and of formulae
  - The linear vs. branching time dichotomy is usually not significant for real-time
    - linear time is almost invariably preferred
  - A different attribute of time that becomes relevant in quantitative models is discrete vs. dense time

Discrete vs. dense (continuous) time

- Discrete time
  - sequence of isolated "steps"
  - every instant has a unique successor
  - e.g.: the naturals $\mathbb{N} = \{0, 1, 2, ...\}$

  + simple and intuitive
  + verification usually decidable (and acceptably complex)
  + robust and elegant theoretical framework

  – cannot express true asynchrony
  – unsuitable to model physical variables

- Dense time
  - arbitrarily small distances
  - the successor of an instant is not defined
  - e.g.: the reals $\mathbb{R}$

  + can model true asynchrony
  + accurate modeling of physical variables

  – tricky to understand
  – verification easily undecidable (or highly complex)
  – lacks a unifying framework

- merely dense vs. continuous is usually not as relevant
  - e.g.: $\mathbb{Q}$ vs. $\mathbb{R}$

Dense Real-time Model-Checking

Timed Automata and
Metric Temporal Logic

Dense Real-time Model-Checking

Dense real-time model checking considers the same model as discrete real-time model checking but with $\mathbb{R}_{\geq 0}$ as time domain:

- A dense Timed Automaton (TA) models the system
- Dense-time Metric Temporal Logic (MTL) models the property

- The syntax of TA and MTL need not be changed for dense time
  - with the possible exception of allowing fractional time bounds
- The semantics of TA and MTL is also unchanged except that:
  - $\mathbb{R}_{\geq 0}$ replaces $\mathbb{N}$ as time domain
  - Infinite words are considered by default:
    - This is a technicality that we will ignore in the presentation for simplicity, although it does affect some results.
      See later for the details.

Dense Real-time Model-Checking

Dense real-time model checking extends standard "untimed" model checking:

- The Timed Automaton (TA) extends the Finite-State Automaton (FSA)
- Metric Temporal Logic (MTL) extends Linear Temporal Logic (LTL)

The Dense Real-time Model Checking problem:

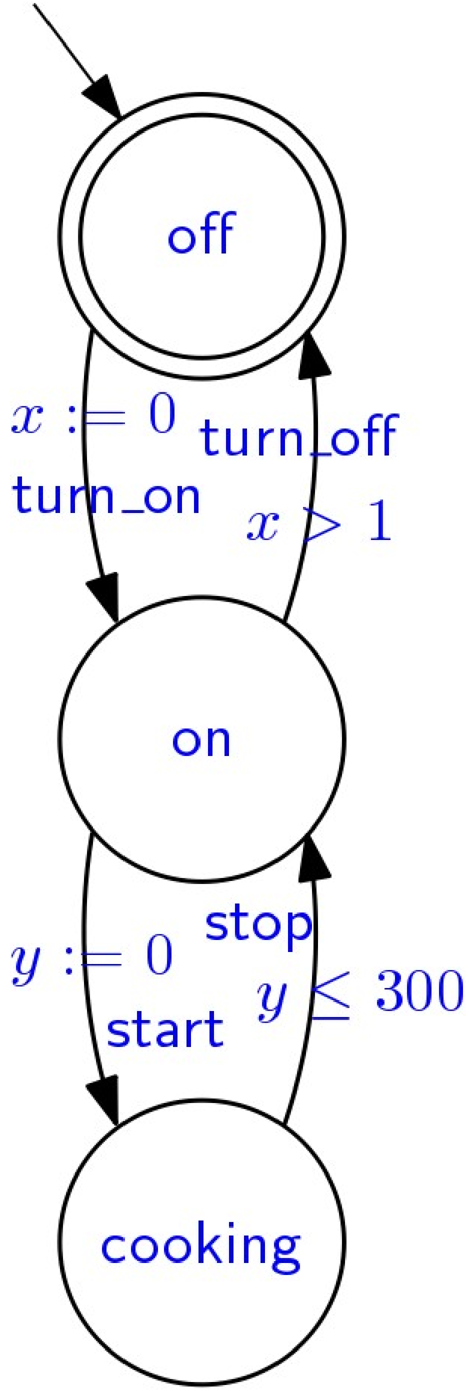
- Given: a dense TA A and an MTL formula F
- Determine: if every run of A satisfies F or not
  - if not, also provide a counterexample: a run of A where F does not hold

A: a TA

F: an MTL formula

$$A \overset{?}{\models} F$$

Timed Automata: Syntax



Timed Automata: Syntax

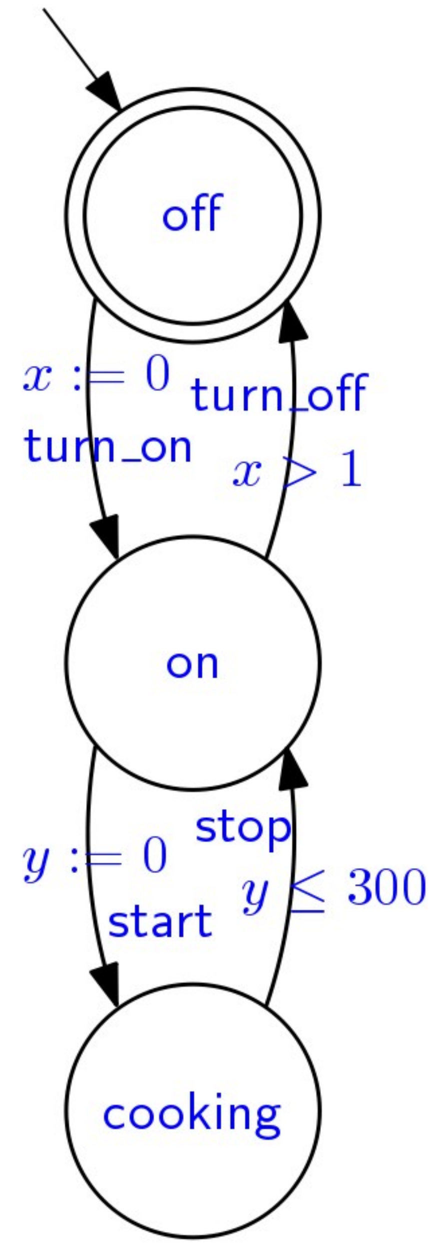
Def. Nondeterministic Timed Automaton (TA):

a tuple [Σ, S, C, I, E, F]:

- Σ: finite nonempty (input) alphabet
- S: finite nonempty set of locations (i.e., discrete states)
- C: finite set of clocks
- I, F: set of initial/final states
- E: finite set of edges [s, σ, c, ρ, s']
  - s ∈ S: source location
  - s' ∈ S: target location
  - σ ∈ Σ: input character (also "label")
  - c: clock constraint in the form:

    c ::= x ≈ k | x – y ≈ k | ¬ c | c1 ∧ c2

    - x, y ∈ C are clocks
    - k ∈ ℤ is an integer constant
    - ≈ is a comparison operator among <, ≤, >, ≥, =
  - ρ ⊆ C: set of clock that are reset (to 0)



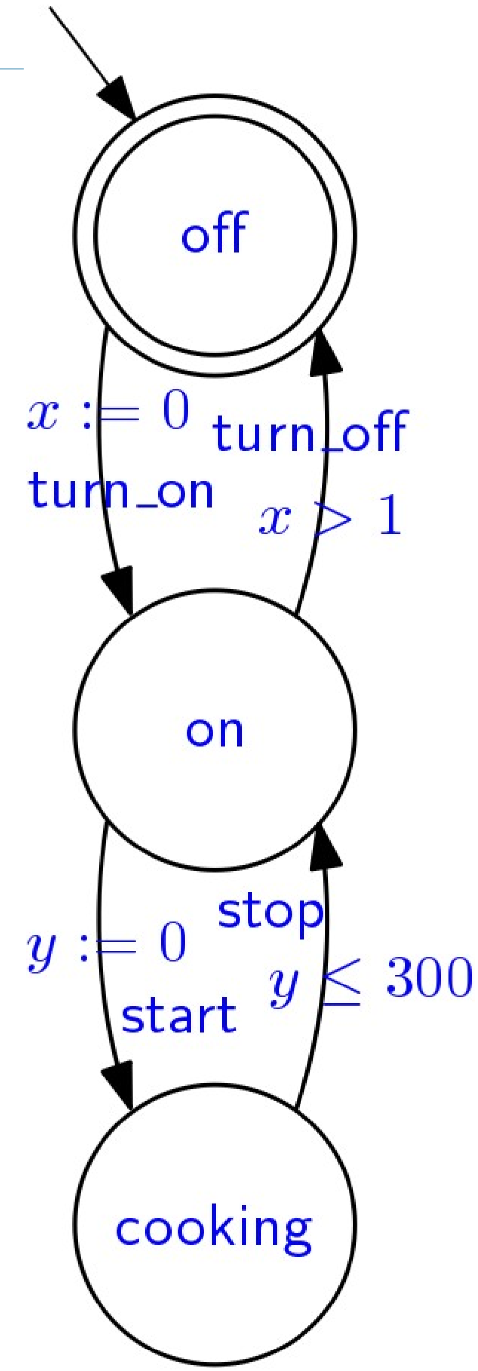
Timed Automata: Semantics

- Accepting run:

  r = [off, (x=0, y=0)]
  [on, (x=0, y=3.2)]
  [cooking, (x=8.5, y=0)]
  [on, (x=81.7, y=73.2)]
  [off, (x=84.91, y=76.41)]

- Over input timed word:

  w = [turn_on, 3.2]
  [start, 11.7]
  [stop, 84.9]
  [turn_off, 88.11]



Timed Automata: Semantics

Def. A timed word $w = w(1)\ w(2) \ldots w(n) \in (\Sigma \times \mathbb{R})^*$ is a sequence of pairs $[\sigma(i), t(i)]$ such that:

- the sequence of timestamps $t(1), t(2), \ldots, t(n)$ is increasing

- $[\sigma(i), t(i)]$ represents the i-th character $\sigma(i)$ read at time $t(i)$

Def. An accepting run of a TA $A=[\Sigma, S, C, I, E, F]$ over input timed word $w = [\sigma(1), t(1)] \ldots [\sigma(n), t(n)] \in (\Sigma \times \mathbb{R})^*$ is a sequence $r = [s(0), v(0,1), \ldots, v(0,|C|)] \ldots [s(n), v(n,1), \ldots, v(n,|C|)]$ $\in (S \times \mathbb{R}^{|C|})^*$ of (extended) states such that:

- it starts from an initial state and ends in an accepting state: $s(0) \in I$ and $s(n) \in F$
- initially all clocks are reset to 0: $v(0,k) = 0$ for all $1 \leq k \leq |C|$
- for every transition $(0 \leq i < n)$:

  $[\ s(i)\ v(i,1) \ldots v(i,|C|)\ ] \dashrightarrow [\ s(i+1)\ v(i+1,1) \ldots v(i+1,|C|)\ ]$

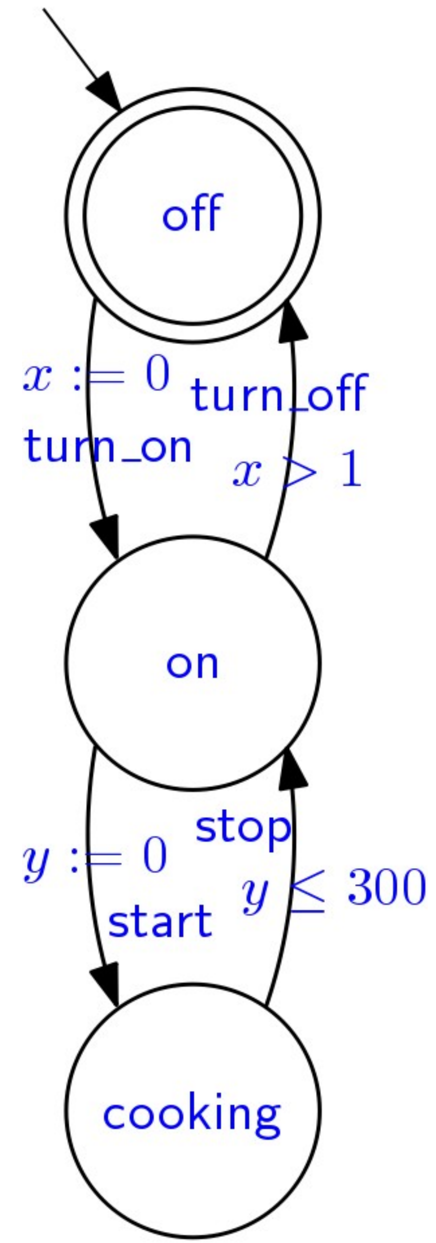
  some edge $[s(i), \sigma(i+1), c, \rho, s(i+1)]$ in E is followed:

  - the clock values $v(i,1) + (t(i+1) - t(i)) \ldots v(i,|C|) + (t(i+1) - t(i))$ satisfy the constraint c
  - $v(i+1,k)$ = if k-th clock is in ρ then 0 else $v(i,k) + t(i+1) - t(i)$

Timed Automata: Semantics

Def. Any TA A=[Σ, S, C, I, E, F] defines
a set of input timed words ⟨A⟩:
$\langle A \rangle \triangleq \{ w \in (\Sigma \times \mathbb{R})^* \mid$ there is an accepting run of A over w }

⟨A⟩ is called the language of A



With regular expressions and arithmetic:

$\langle A \rangle$ = ([turn_on, t_1] ([start, t_2] [stop, t_3])* [turn_off, t_4])*

with $t_3 - t_2 \le 300$ and $t_4 - t_1 > 1$

Metric (Linear) Temporal Logic

$\Diamond_{[2,4)}$ stop

"there is an occurrence of stop between 2 (included) and 4 (excluded) time units in the future"

- [any, t < 2]* [stop, 2] [stop, 3] [any, 3.5] [any, 3.7] ...
- [any, t < 3.99]* [stop, 3.99] [any, 4] [any, t > 4] ...

$\Box_{(2,4]}$ start

"start holds between 2 (excluded) and 4 (included) time units in the future"

- [any, t ≤ 2] [start, 2.2] [start, 3] [start, 4] [any, t > 4] ...
- [any, t ≤ 2] [start, 4] [any, t > 4] ...
- [stop, 0] [stop, 0.3] [stop, 0.7]

Metric (Linear) Temporal Logic

$\Box$ (start $\Rightarrow$ $\Diamond$(3,10] stop)

"every occurrence of start is followed by an occurrence of stop between 3 (excluded) and 10 (included) time units in the future"

cook U(3,10] stop

"stop occurs between 3 (excluded) and 10 (included) time units in the future, and cook holds until then"

Metric (Linear) Temporal Logic: Syntax

Def. Propositional Metric Temporal Logic (MTL) formulae are defined by the grammar:

$$F ::= p \mid \neg F \mid F \wedge G \mid F\ U\langle a,b\rangle\ G$$

with $p \in P$ any atomic proposition and $\langle a,b\rangle$ is an interval of the time domain (w.l.o.g. with integer endpoints).

Temporal (modal) operators:

- next: $X\ F \triangleq True\ U[1,1]\ F$
- bounded until: $F\ U\langle a,b\rangle\ G$
- bounded release: $F\ R\langle a,b\rangle\ G \triangleq \neg(\neg F\ U\langle a,b\rangle\ \neg G)$
- bounded eventually: $\Diamond\langle a,b\rangle\ F \triangleq True\ U\langle a,b\rangle\ F$
- bounded always: $\Box\langle a,b\rangle\ F \triangleq \neg \Diamond\langle a,b\rangle\ \neg F$
- intervals can be unbounded; e.g., $[3, \infty)$
- intervals with pseudo-arithmetic expressions, e.g.:
  - ≥ 3 for $[3, \infty)$
  - $= 1$ for $[1,1]$
  - $[0, \infty)$ is simply omitted

$$\Box\ (\ start \Rightarrow \Diamond(3,10]\ stop\ )$$

Metric Temporal Logic: Semantics

Def. A timed word $w = [\sigma(1), t(1)]\,[\sigma(2), t(2)] \ldots [\sigma(n), t(n)] \in (P \times \mathbb{R})^*$ satisfies an LTL formula F at position $1 \leq i \leq n$, denoted $w, i \models F$, under the following conditions:

- $w, i \models p$ iff $p = \sigma(i)$
- $w, i \models \neg F$ iff $w, i \models F$ does not hold
- $w, i \models F \wedge G$ iff both $w, i \models F$ and $w, i \models G$ hold
- $w, i \models F\ U\langle a,b\rangle\ G$ iff for some $i \leq j \leq n$ such that $t(j) - t(i) \in \langle a,b\rangle$ it is: $w, j \models G$ and for all $i \leq k < j$ it is $w, k \models F$
  - i.e., F holds until G will hold within ‹a, b›

For derived operators:

- $w, i \models \Diamond\langle a,b\rangle F$ iff for some $i \leq j \leq n$ such that $t(j) - t(i) \in \langle a,b\rangle$ it is: $w, j \models F$
  - i.e., F holds eventually within ‹a,b›
- $w, i \models \Box\langle a,b\rangle F$ iff for all $i \leq j \leq n$ such that $t(j) - t(i) \in \langle a,b\rangle$ it is: $w, j \models F$
  - i.e., F holds always within ‹a,b›

Metric Temporal Logic: Semantics

Def. Satisfaction:

$$w \models F \quad \triangleq \quad w, 1 \models F$$

i.e., timed word w satisfies formula F initially

Def. Any MTL formula F defines a set of timed words ⟨F⟩:

$$\langle F \rangle \triangleq \{ w \in (P \times \mathbb{R})^* \mid w \models F \}$$

⟨F⟩ is called the language of F

Dense Real-time Model-Checking

What's Decidable?

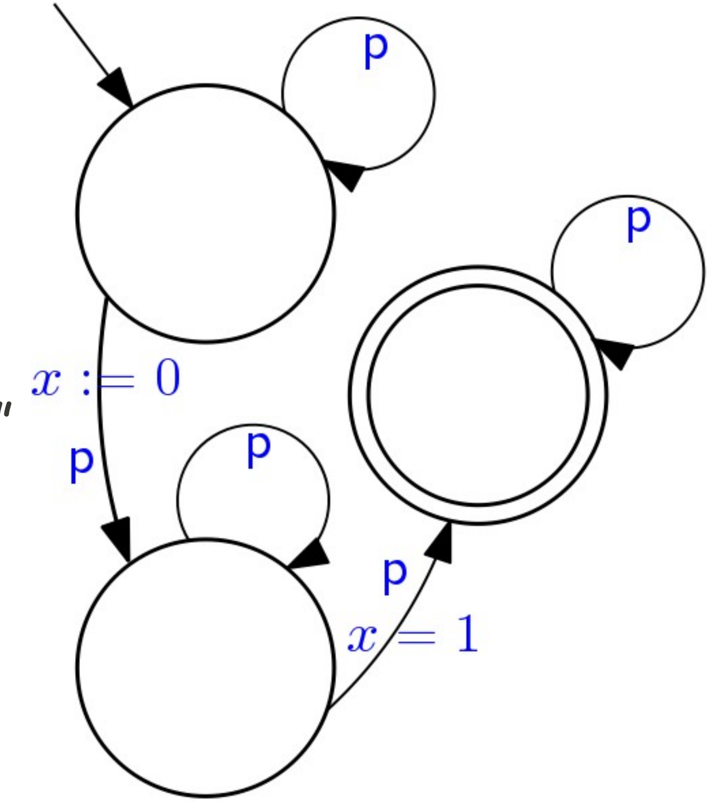
TAs not Closed under Complement

A: a dense TA F: a dense-time MTL formula

$$A \overset{?}{\models} F$$

Fundamental problem:

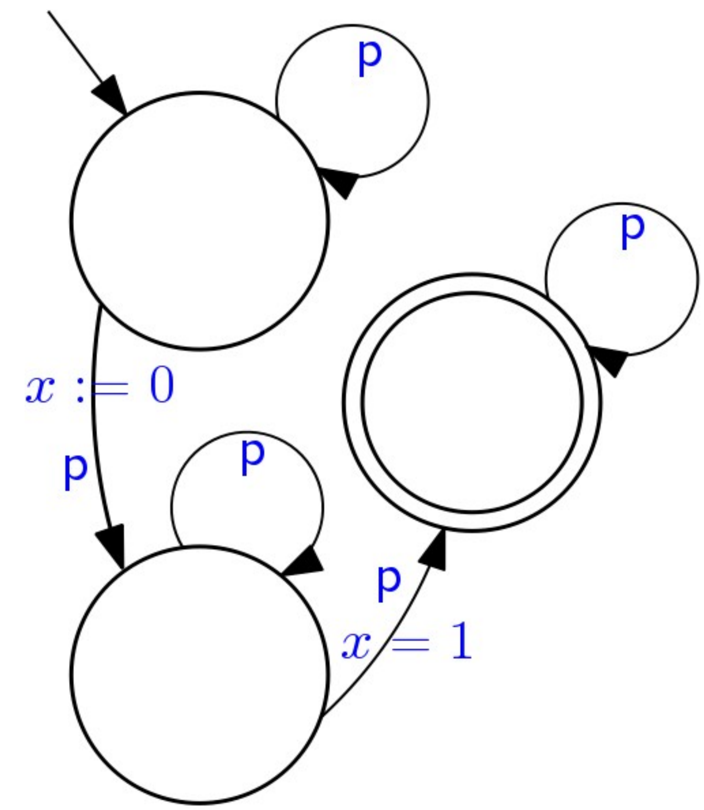
- Dense timed automata are not closed under complement
  - The complement of the language of this TA isn't accepted by any TA:
    - language of this TA: "there exist two p's separated by one t.u."
    - complement language: "no two p's are separated by one t.u."
    - intuition: need a clock for each p within the past time unit, but there can be an unbounded number of such p's because time is dense



TAs not Closed under Complement

Fundamental problem:

- Dense TAs are not closed under complement
- MTL is clearly closed under complement
  - Language of the TA: $\Diamond (p \wedge \Diamond_{=1} p)$
  - Complement language of the TA:
    $\neg \Diamond (p \wedge \Diamond_{=1} p) = \Box (p \Rightarrow \neg \Diamond_{=1} p)$
- Hence, automata-theoretic dense real-time model-checking is unfeasible



Dense MTL Model Checking is Undecidable

An even more fundamental problem:

- The dense-time model-checking problem for MTL and TAs is undecidable (for infinite words)
  - no approach is going to work, not just the automata-theoretic one
- MTL and TAs are "too expressive" over dense time

What's Decidable about Timed Automata

Let's revisit the three algorithmic components of automata-theoretic model checking:

- MTL2TA: given MTL formula F build TA a(F) such that ⟨F⟩ = ⟨a(F)⟩
  - undecidable problem (for infinite words)
- TA-Intersection: given TAs A, B build TA C such that ⟨A⟩ ∩ ⟨B⟩ = ⟨C⟩
  - decidable
- TA-Emptiness: given TA A check whether ⟨A⟩ = ∅ is the case
  - decidable!

Dense Real-time Model-Checking

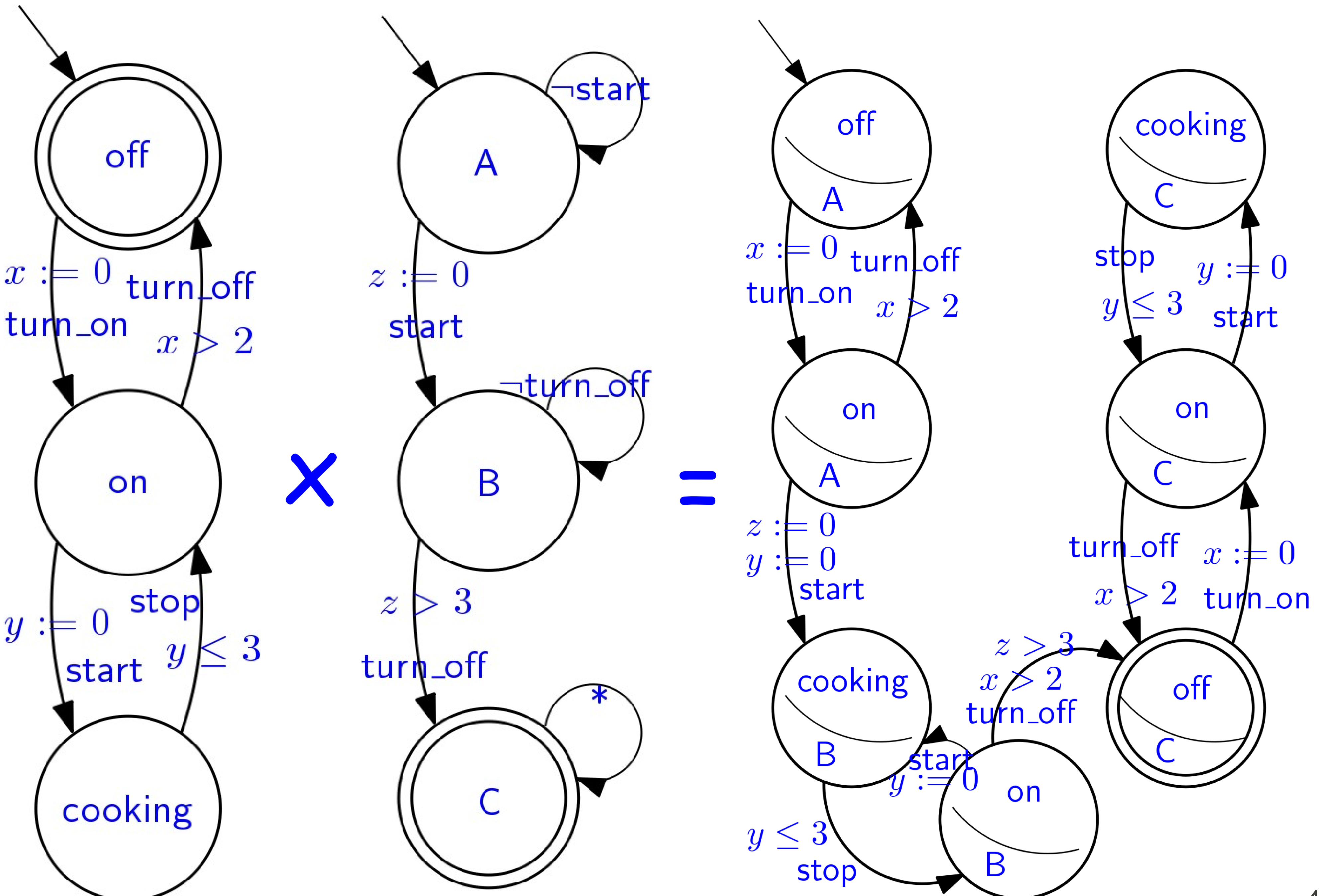
Intersection of Timed Automata

TA-Intersection: running TAs in parallel

Given TAs A, B it is always possible to build automatically a TA C that accepts precisely the words that both A and B accept.

TA C represents all possible parallel runs of A and B where a timed word is accepted if and only if both A and B would accept it. The construction is called "product automaton".

TA-Intersection: Example



TA-Intersection: running TAs in parallel

Def. Given TAs $A=[\Sigma, S^A, C^A, I^A, E^A, F^A]$ and $B=[\Sigma, S^B, C^B, I^B, E^B, F^B]$

let $C \triangleq A \times B \triangleq [\Sigma, S^C, C^C, I^C, E^C, F^C]$ be defined as:

- $S^C \triangleq S^A \times S^B$
- $C^C \triangleq C^A \cup C^B$ (assuming w.l.o.g. that they are disjoint sets)
- $I^C \triangleq \{ (s, t) \mid s \in I^A$ and $t \in I^B \}$
- $[(s, t), \sigma, c^A \wedge c^B, \rho^A \cup \rho^B, (s', t')] \in E^C$ iff
  $[s, \sigma, c^A, \rho^A, s'] \in E^A$ and $[t, \sigma, c^B, \rho^B, t'] \in E^B$
- $F^C \triangleq \{ (s, t) \mid s \in F^A$ and $t \in F^B \}$

Theorem.

$$\langle A \times B \rangle = \langle A \rangle \cap \langle B \rangle$$

Dense Real-time Model-Checking

Checking the Emptiness of Timed Automata

TA-Emptiness

Given a TA *A* it is always possible to check automatically if it accepts some timed word.

Outline of the algorithm:

- Assume that clock constraints involve integer constants only
  - this is without loss of generality as it amounts to scaling
- Define an equivalence relation over extended states
  - an extended state is a tuple [s, v(1), ..., v(|C|)] with a location s and a value v(i) for every clock in C.
- All extended states in the same equivalence class are equivalent w.r.t. satisfaction of clock constraints
- The equivalence relation is such that there is a finite number of equivalence classes for any given TA
- Given a TA *A*, build an FSA reg(*A*) – the "region automaton":
  - the states of reg(*A*) represent the equivalence classes of the extended states of any run of of *A*
  - the edges of reg(*A*) represent evolution of any extended state within the equivalence class over any run of *A*
- Checking the emptiness of reg(*A*) is equivalent to checking the emptiness of *A*

Integer vs. Rational vs. Irrational

- The domain for time is $\mathbb{R}_{\geq 0}$
- What about the domain for time constraints?
  - constants in clock constraints of TAs (e.g.: $x < k$)

1. Same as the domain for time: $\mathbb{R}_{\geq 0}$
   - e.g.: $x < \pi$
   - emptiness becomes undecidable!
2. Discrete time domain: integers $\mathbb{N}$
   - e.g.: $x < 5$
   - emptiness fully decidable (see algorithm next)
3. Dense but not continuous: rationals $\mathbb{Q}_{\geq 0}$
   - e.g.: $x < 1/3$
   - emptiness is reducible to the integer case

Integer vs. Rational

- Dense but not continuous: rationals $\mathbb{Q}_{\geq 0}$
  - Let A be a TA with rational constants
    - let m be the least common multiple of denominators of all constants appearing in the clock constraints of A
    - let A*m be the TA obtained from A by multiplying every constants in the clock constraints by m
      - A*m has only integers constants in its clock constraints
  - A accepts any timed word

    $[\sigma(1), t(1)]\ [\sigma(2), t(2)] \ldots [\sigma(n), t(n)]$

    iff A*m accepts the "scaled" timed word

    $[\sigma(1), m*t(1)]\ [\sigma(2), m*t(2)] \ldots [\sigma(n), m*t(n)]$
  - Hence checking the emptiness of A*m is equivalent to checking the emptiness of A

Equivalence Relation over Extended States

Let us fix a TA $A = [\Sigma, S, C, I, E, F]$ with $C = [x(1), ..., x(n)]$

- For any clock x(i) in C let M(i) be the largest constant involving clock x(i) in any clock constraint in E
- Let $[v(1), ..., v(n)] \in \mathbb{R}_{\geq 0}{}^{n}$ denote a "clock evaluation" representing any assignment of values to clocks
- Equivalence of two clock evaluations:
  $[v(1), ..., v(n)] \sim [v'(1), ..., v'(n)]$ iff all of the following hold:
  1. For all $1 \leq i \leq n$: $int(v(i)) = int(v'(i))$ or $v(i), v'(i) > M(i)$
  2. For all $1 \leq i,j \leq n$ such that $v(i) \leq M(i)$ and $v(j) \leq M(j)$:
     $frac(v(i)) \leq frac(v(j))$ iff $frac(v'(i)) \leq frac(v'(j))$
  3. For all $1 \leq i \leq n$ such that $v(i) \leq M(i)$:
     $frac(v(i)) = 0$ iff $frac(v'(i)) = 0$
- Note: int(x) is the integer part of x; frac(x) is the fractional part of x

Clock Regions

Def. A clock region is an equivalence class of clock evaluations induced by the equivalence relation ~

- For a clock evaluation $v = [v(1), \ldots, v(n)] \in \mathbb{R}_{\geq 0}^n$, [[v]] denotes the clock region v belongs to
- As a consequence of the definition of ~, any clock region can be uniquely characterized by a finite set of constraints on clocks
  - v = [0.4; 0.9; 0.7; 0] for 4 clocks w, x, y, z
  - [[v]] is $z = 0 < w < y < x < 1$
- Fact: clock regions are always in finite number

Clock Regions (cont'd)

More systematically:

- given a set of clocks $C = [x(1), \ldots, x(n)]$
- with $M(i)$ the largest constant appearing in constraints on clock $x(i)$

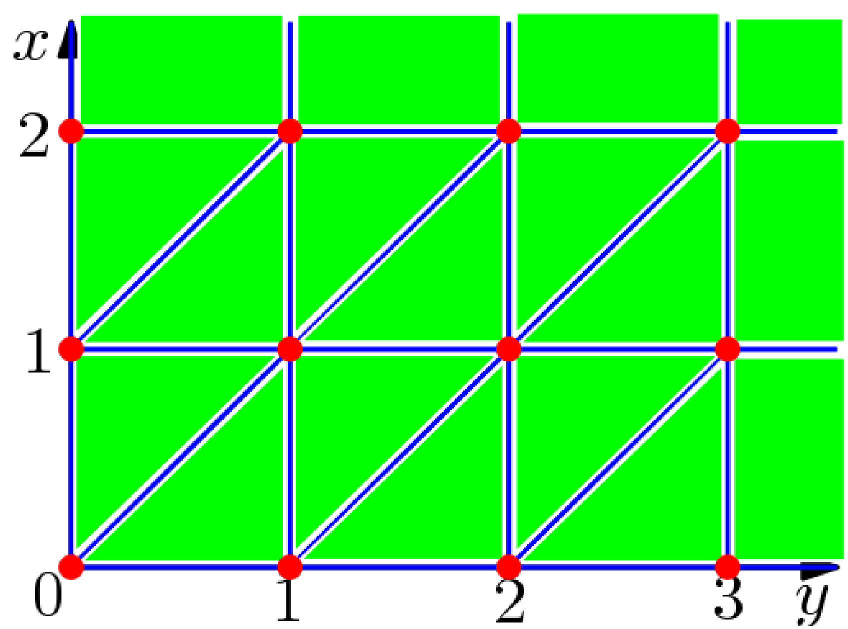
a clock region is uniquely characterized by

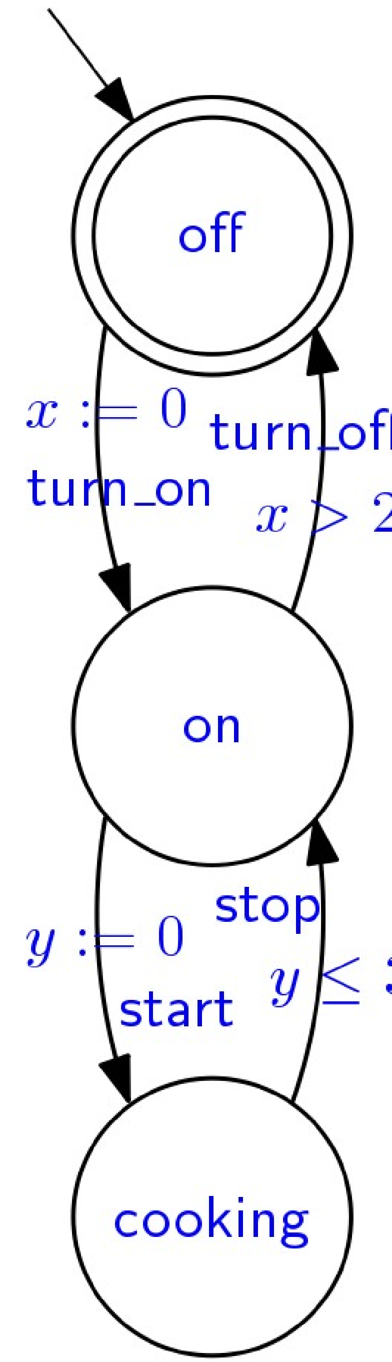
- For each clock $x(i)$ a constraint in the form:
  - $x(i) = c$ with $c = 0, 1, \ldots, M(i)$; or
  - $c - 1 < x(i) < c$ with $c = 1, \ldots, M(i)$; or
  - $x(i) > M(i)$
- For each pair of clocks $x(i)$, $x(j)$ a constraint in the form
  - $frac(x(i)) < frac(x(j))$
  - $frac(x(i)) = frac(x(j))$
  - $frac(x(i)) > frac(x(j))$

  (These are unnecessary if $x(i) = c$, $x(j) = c$, $x(i) > M(i)$, or $x(j) > M(j)$)

Clock Regions: Example

- Clocks C = [x, y]
- M(x) = 2; M(y) = 3
- All 60 possible clock regions:
  - 12 corner points
  - 30 open line segments
  - 18 open regions





Time-successors of Regions

- Fact: a clock evaluation v satisfies a clock constraint c iff any other clock evaluation in [[v]] satisfies c
  - Hence, we can say that a "clock region satisfies a clock constraint"

Def. The time successor time-succ(R) of a clock region R is the set of all clock regions (including R itself) that can be reached from R by letting time pass (i.e., without resetting any clock).

Given a clock region R it is always possible to compute all other clock regions that can be reached from R by letting time pass (i.e., without resetting any clock)

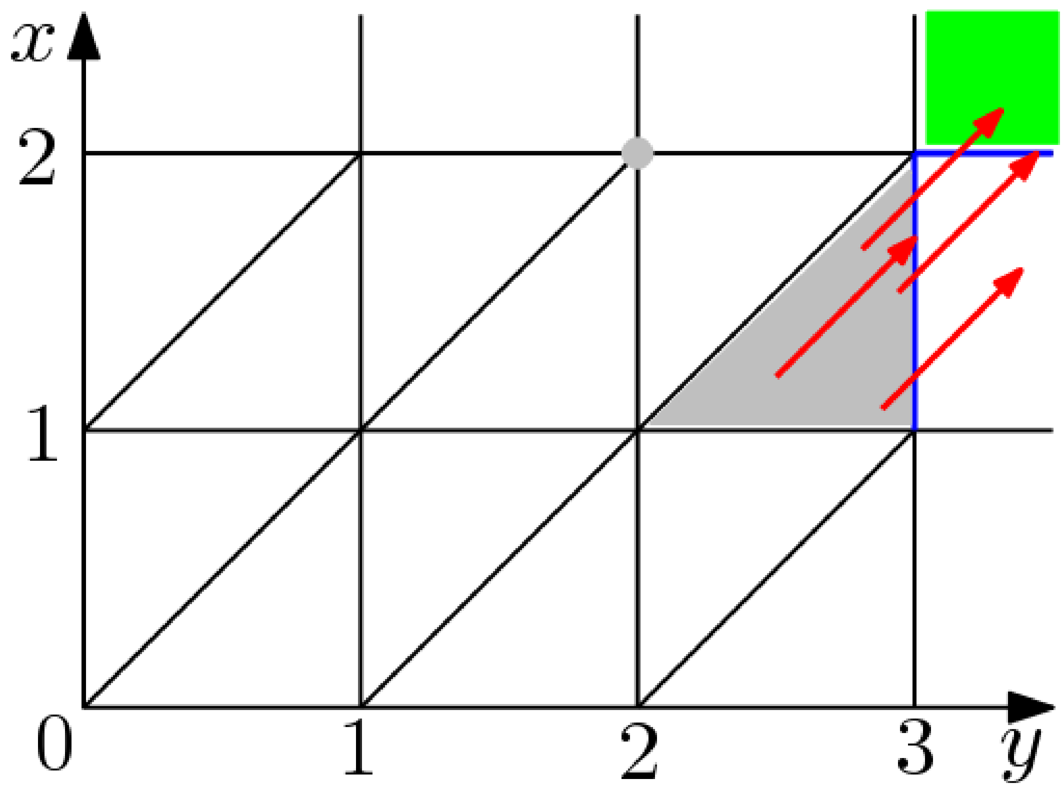
- Graphically:
  - the time-successors of a region R are the regions that can be reached by moving along a line parallel to the diagonal in the upward direction, starting from any point in R

(For a precise definition see e.g.: Alur & Dill, 1994)

Time-successors of Regions: Example

- Graphically:
  - the time-successors of a region R are the regions that can be reached by moving along a line parallel to the diagonal in the upward direction, starting from any point in R

- Example:
  - successors of region
    2 < y < 3; 1 < x < y-1
    (other than the region itself):
    - y > 3; 1 < x < 2
    - y > 3; x = 2
    - y = 3; 1 < x < 2
    - y > 3; x > 2
  - successors of region
    y = 1; x = 2
    (other than the region itself):
    - 2 < y < 3; x > 2
    - ...



Region Automaton Construction

For a timed automaton A it is always possible to build an FSA $reg(A)$ (the "region automaton" of A) such that:

$$\langle A \rangle = \emptyset \quad \text{iff} \quad \langle reg(A) \rangle = \emptyset$$

Def. Given a TA $A = [\Sigma, S, C, I, E, F]$ its region automaton $reg(A) \triangleq [\Sigma, rS, rI, rE, rF]$ is defined as:

- $rS \triangleq \{ (s, r) \mid s \in S$ and r is a clock region $\}$
- $rI \triangleq \{ (s, [[0, 0, \ldots, 0]]) \mid s \in I \}$

  _the clock region where all clocks are reset to 0
- $rE(\sigma, [s, r]) \triangleq \{ (s', r') \mid [s, \sigma, c, \rho, s'] \in E$ and there exists a region $r'' \in$ time-succ(r) such that r'' satisfies c, and r' is obtained from r'' by resetting all clocks in ρ to 0 $\}$
- $rF \triangleq \{ (s, r) \mid s \in F \}$

Region Automaton: Example