ETH Zürich
Chair of Software Engineering

Concurrent Object-Oriented Programming

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Lecture 11: An introduction to CSP

CSP: Origin

Communicating Sequential Processes: C.A.R. Hoare

1978 paper, based in part on ideas of E.W. Dijkstra (guarded commands, 1978 paper and "A Discipline of Programming" book)

Revised with help of S. D. Brooks and A.W. Roscoe

1985 book, revised 2004

Complete reference: *The Theory and Practice of Concurrency*, A. W. Roscoe, Prentice Hall 1997 (2005) (used extensively in the present slides)

CSP purpose

Concurrency formalism

- Expresses many concurrent situations elegantly
- Influenced design of several concurrent programming languages, in particular Occam (Transputer)

Calculus

- Formally specified: laws
- Makes it possible to prove properties of systems

Traces

A trace is a sequence of events, for example

<coin, coffee, coin, coffee>

Many traces of interest are infinite, for example

<coin, coffee, coin, coffee, …>

(Can be defined formally, e.g by regular expressions, but such traces definition are not part of CSP; they are descriptions of CSP process properties.)

Events come from an *alphabet*. The alphabet of all possible events is written Σ in the following.

Processes and their traces

A CSP process is characterized (although not necessarily defined fully) by the set of its traces. For example a process may have the trace set

{<>,
<coin, coffee>,
<coin, tea>}

The special process STOP has a trace set consisting of a single, empty trace:

{<>}

Basic CSP syntax

$P ::=$

$STOP$	|	-- Does not engage in any events
$a \rightarrow Q$	|	-- Engages in a, then acts like Q
$Q \sqcap R$	|	-- Internal choice
$Q \,\Box\, R$	|	-- External choice
$Q \underset{E}{\parallel} R$	|	-- Concurrency (E: subset of alphabet)
$Q \parallel R$	|	-- Lock-step concurrency (same as $Q \underset{\Sigma}{\parallel} R$)
$Q \setminus E$	|	-- Hiding
$\mu Q \bullet f(Q)$		-- Recursion

Generalization of → notation

Basic:

$a \rightarrow P$

Generalization:

$x: E \rightarrow P(x)$

Accepts any event from E, then executes $P(x)$ where x is that event

Also written

$?\, x: E \rightarrow P(x)$

Some laws of concurrency

1. $P \,||\, Q = Q \,||\, P$
2. $P \,||\, (Q \,||\, R)) = ((P \,||\, Q) \,||\, R)$

3. $P \,||\, STOP = STOP$

4. $(c \rightarrow P) \,||\, (c \rightarrow Q) = (c \rightarrow (P \,||\, Q))$
5. $(c \rightarrow P) \,||\, (d \rightarrow Q) = STOP$ -- If $c \neq d$

6. $(x{:}\, A \rightarrow P\,(x)) \,||\, (y{:}\, B \rightarrow Q\,(y)) =$
   $(z{:}\, (A \cap B) \rightarrow (P\,(z) \,||\, Q\,(z))$

Basic notions

Processes engage in events

Example of basic notation:

$CVM = (coin \rightarrow coffee \rightarrow coin \rightarrow coffee \rightarrow STOP)$

Right associativity: the above is an abbreviation for

$CVM = (coin \rightarrow (coffee \rightarrow (coin \rightarrow (coffee \rightarrow STOP))))$

Trace set of CVM: {<coin, coffee, coin, coffee>}

The events of a process are taken from its alphabet:

$\alpha(CVM) = \{coin, coffee\}$

STOP can engage in no events

Traces

$$traces\ (e \rightarrow P) = \{\langle e \rangle + s \quad | \quad s \in traces\ (P)\}$$

Exercises: determine traces

P ::=

STOP	| -- Does not engage in any events
$a \rightarrow Q$	| -- Engages in a, then acts like Q
$Q \sqcap R$	| -- Internal choice
$Q \,\square\, R$	| -- External choice
$Q \underset{E}{\parallel} R$	| -- Concurrency (E: subset of alphabet)
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$\mu Q \bullet f(Q)$	-- Recursion

Recursion

$CLOCK = (tick \rightarrow CLOCK)$

This is an abbreviation for

$$CLOCK = \mu P \bullet (tick \rightarrow P)$$

A recursive definition is a fixpoint equation. The μ notation denotes the fixpoint

Accepting one of a set of events; channels

Basic notation:

$$? \, x: A \rightarrow P\,(x)$$

Accepts any event from A, then executes $P\,(x)$ where x is that event

Channel names

Example:

$$? \, y: c.A \rightarrow d.y'$$

(where $c.A$ denotes $\{c.x \mid x \in A\}$ and y' denotes y deprived of its initial channel name, e.g. $(c.a)' = a$)

More convenient notation for such cases involving channels:

$$\boxed{c? \, x: A \rightarrow d!x}$$

A simple buffer

$$COPY = c?\,x{:}\,A \rightarrow d!x \rightarrow COPY$$

External choice

$$COPYBIT = (in.0 \rightarrow out.0 \rightarrow COPYBIT \;\square\; in.1 \rightarrow out.1 \rightarrow COPYBIT)$$

External choice

$COPY1 = in?\, x{:}\, A \rightarrow out1!x \rightarrow COPY1$

$COPY2 = in?\, x{:}\, B \rightarrow out2!x \rightarrow COPY2$

$COPY3 = COPY1 \,\square\, COPY2$

External choice

Consider

CHM1 = (in1f → out50rp → out20rp → out20rp → out10rp)

CHM2 = (in1f → out50rp → out50rp)

CHM = CHM1 □ CHM2

Lock-step concurrency

Consider

$P \quad = \quad ?x{:}\, A \to P'$

$Q \quad = \quad ?x{:}\, B \to Q'$

Then

$P \,||\, Q = ?\, x \to$

- $(P' \,||\, Q')$ if $x \in A \cap B$
- $STOP$ otherwise

(to be generalized soon)

More examples

$$
\begin{aligned}
VMC = \\
&(in2f \rightarrow \\
&\qquad ((large \rightarrow VMC) \;\Box \\
&\qquad (small \rightarrow out1f \rightarrow VMC)) \\
&\Box \\
&(in1f \rightarrow \\
&\qquad ((small \rightarrow VMC) \;\Box \\
&\qquad (in1f \rightarrow large \rightarrow VMC))
\end{aligned}
$$

$$
\begin{aligned}
FOOLCUST = (&in2f \rightarrow large \rightarrow FOOLCUST \;\Box \\
&in1f \rightarrow large \rightarrow FOOLCUST)
\end{aligned}
$$

$$
\begin{aligned}
FV = FOOLCUST \,||\, VMC = \\
&\mu P \bullet (in2f \rightarrow large \rightarrow FV \;\Box\; in1f \rightarrow STOP)
\end{aligned}
$$

Hiding

Consider

$$P = a \rightarrow b \rightarrow Q$$

Assuming Q does not involve b, then

$$P \setminus \{b\} = a \rightarrow Q$$

More generally:

$(a \rightarrow P) \setminus E =$

- $P \setminus E$ if $a \in E$
- $a \rightarrow (P \setminus E)$ if $a \notin E$

Hiding introduces internal non-determinism

Consider

$$R = (a \rightarrow P) \;\Box\; (b \rightarrow Q)$$

Then

$$R \setminus \{a, b\} = P \sqcap Q$$

Internal non-deterministic choice

CH1F = (in1f →

((out20rp → out20rp →

out20rp →out20rp →out20rp → CH1F)

⊓

(out50rp → out50rp → CH1F)))

Non-deterministic internal choice: another application

$$
\begin{array}{lll}
\text{TRANSMIT}(x) & = & in?x \rightarrow \text{LOSSY}(x) \\
 & & \\
\text{LOSSY}(x) & = & \\
 & & out!x \rightarrow \text{TRANSMIT}(x) \\
 & \sqcap & out!x \rightarrow \text{LOSSY}(x) \\
 & \sqcap & \text{TRANSMIT}(x)
\end{array}
$$

The general concurrency operator

Consider

$$P = ?x{:}\ A \rightarrow P'$$

$$Q = ?x{:}\ B \rightarrow Q'$$

Then

$$P \underset{E}{||} Q = ?\,x \rightarrow$$

- $P' \underset{E}{||} Q'$ if $x \in E \cap A \cap B$
- $P' \underset{E}{||} Q$ if $x \in A\text{-}B\text{-}E$
- $P \underset{E}{||} Q'$ if $x \in B\text{-}A\text{-}E$
- $(P' \underset{E}{||} Q) \sqcap (P \underset{E}{||} Q')$ if $x \in (A \cap B) - E$

Special cases of concurrency

Lock-step concurrency:

$$P \mid\mid Q \qquad = \; P \mathbin{\mathop{\mid\mid}_{\Sigma}} Q$$

Interleaving:

$$P \mid\mid\mid Q \qquad = \; P \mathbin{\mathop{\mid\mid}_{\varnothing}} Q$$

Lock-step concurrency (reminder)

Consider

$P = ?x: A \rightarrow P'$

$Q = ?x: B \rightarrow Q'$

Then

$P \mid\mid Q = ?\, x \rightarrow$

- $(P' \mid\mid Q')$ if $x \in E \cap A \cap B$
- $STOP$ otherwise

Laws of non-deterministic internal choice

$P \sqcap P = P$

$P \sqcap Q = Q \sqcap P$

$P \sqcap (Q \sqcap R) = (P \sqcap Q) \sqcap R$

$x \rightarrow (P \sqcap Q) = (x \rightarrow P) \sqcap (x \rightarrow Q)$

$P \mid\mid (Q \sqcap R) = (P \mid\mid Q) \sqcap (P \mid\mid R)$

$(P \sqcap Q) \mid\mid R = (P \mid\mid R) \sqcap (Q \mid\mid R)$

The recursion operator is not distributive; consider:

$$P = \mu X \bullet ((a \rightarrow X) \sqcap (b \rightarrow X))$$

$$Q = (\mu X \bullet (a \rightarrow X)) \sqcap (\mu X \bullet (b \rightarrow X))$$

Note on external choice

From previous slide:

$$x \rightarrow (P \sqcap Q) = (x \rightarrow P) \sqcap (x \rightarrow Q)$$

The question was asked in class of whether a similar property also applies to external choice $\Box$

The conjectured property is

$$x \rightarrow (P \Box Q) = (x \rightarrow P) \Box (x \rightarrow Q)$$

It does not hold, since

$$(x \rightarrow P) \Box (x \rightarrow Q) = x \rightarrow (P \sqcap Q)$$

(As a consequence of rule on next page)

General property of external choice

$(?x{:}\ A \rightarrow P) \,\square\, (?x{:}\ B \rightarrow Q) =$

$?x{:}\ A \cup B \rightarrow$

- P if $x \in A\text{-}B$
- Q if $x \in B\text{-}A$
- $P \sqcap Q$ if $x \in A \cap B$

Traces

$$traces\ (e \rightarrow P) = \{\langle e\rangle + s \quad | \quad s \in traces\ (P)\}$$

Exercise: determine traces

P ::=

STOP	| -- Does not engage in any events
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Refinement

Process Q *refines* (specifically, *trace-refines*) process P if

$$\text{traces}(Q) \subseteq \text{traces}(P)$$

For example:

$$P \quad \textit{refines} \quad P \sqcap Q$$

The trace model is not enough

The traces of and are the same:

$$\text{traces } (P \square Q) = \text{traces } (P) \cup \text{traces } (Q)$$

$$\text{traces } (P \sqcap Q) = \text{traces } (P) \cup \text{traces } (Q)$$

But the processes can behave differently if for example:

$$P = a \rightarrow b \rightarrow STOP$$

$$Q = b \rightarrow a \rightarrow STOP$$

Traces define what a process may do, not what it may refuse to do

Refusals

For a process P and a trace t of P:

- An event set $es \in \mathbb{P}(\Sigma)$ is a *refusal set* if P can forever refuse all events in es
- Refusals (P) is the set of P's refusal sets
- Convention: keep only maximal refusal sets

  (if X is a refusal set and $Y \subseteq X$, then Y is a refusal set)

This also leads to a notion of "failure":

- Failures (P, t) is Refusals (P / t)

where P/t is $\boxed{P\ \textit{after}\ t}$:

traces (P / t) = {u | t + u $\in$ traces (p))

Comparing failures

Compare

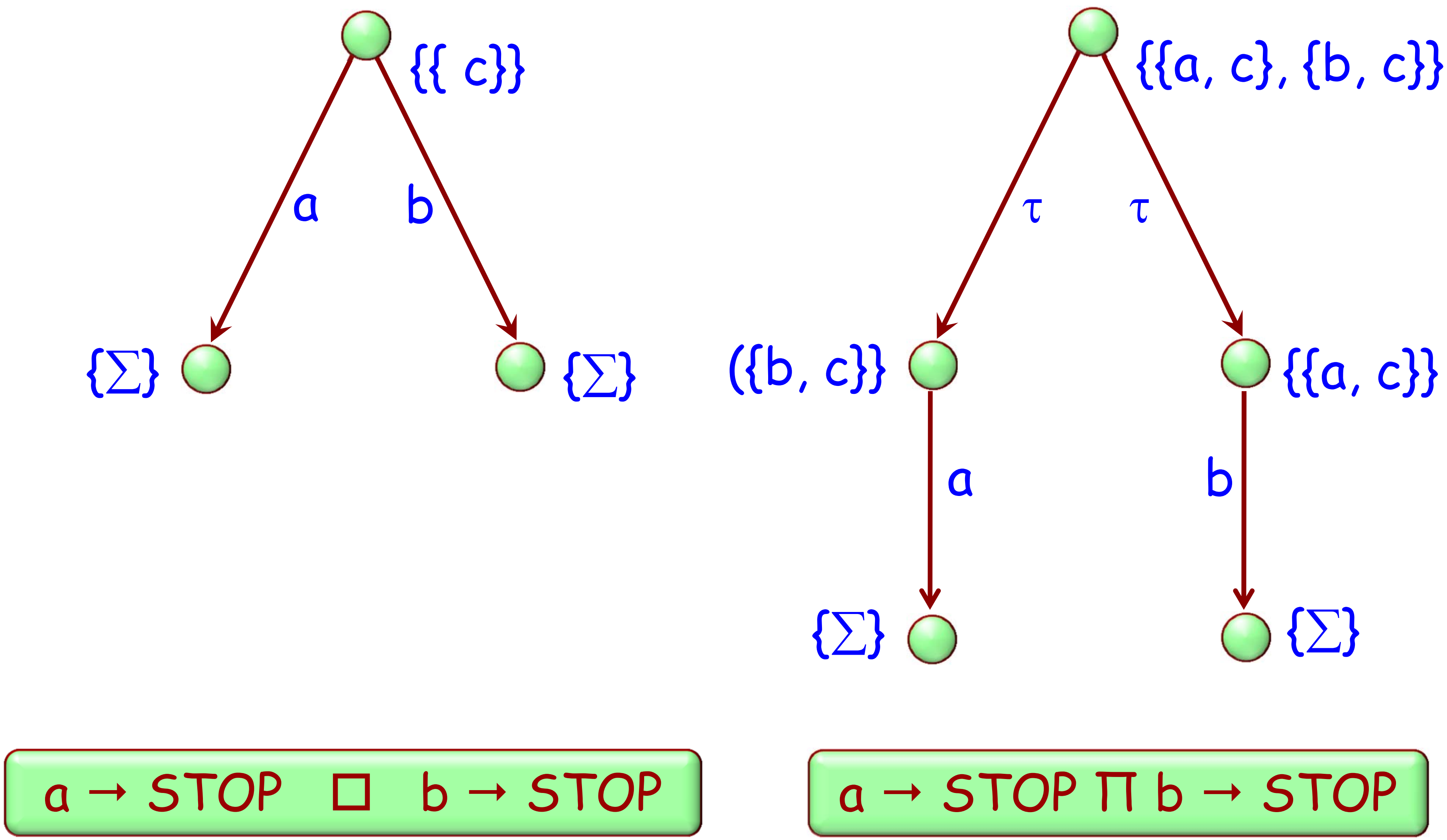
- $P = a \rightarrow STOP \;\Box\; b \rightarrow STOP$
- $Q = a \rightarrow STOP \;\sqcap\; b \rightarrow STOP$

Same traces, but:

- Refusals (P) = ∅
- Refusals (Q) = {{a}, {b}}

Refusal sets (from labeled transition diagram)

$\Sigma = \{ a, b, c\}$



A more complete notion of refinement

Process Q *failures-refines* process P if both

$$traces\ (Q) \subseteq traces\ (P)$$
$$failures\ (Q) \subseteq failures\ (P)$$

Makes it possible to distinguish between $\square$ and $\sqcap$

Divergence

A process diverges if it is not refusing all events but not communicating with the environment

This happens if a process can engage in an infinite sequence of τ transitions

An example of diverging process:

$$(\mu\, p.a \rightarrow p) \setminus a$$

CSP: Summary

A calculus based on mathematical laws

Provides a general model of computation based on communication

Serves both as specification of concurrent systems and as a guide to implementation

One of the most influential models for concurrency work