Software Verification Exercise class: Real Time Systems

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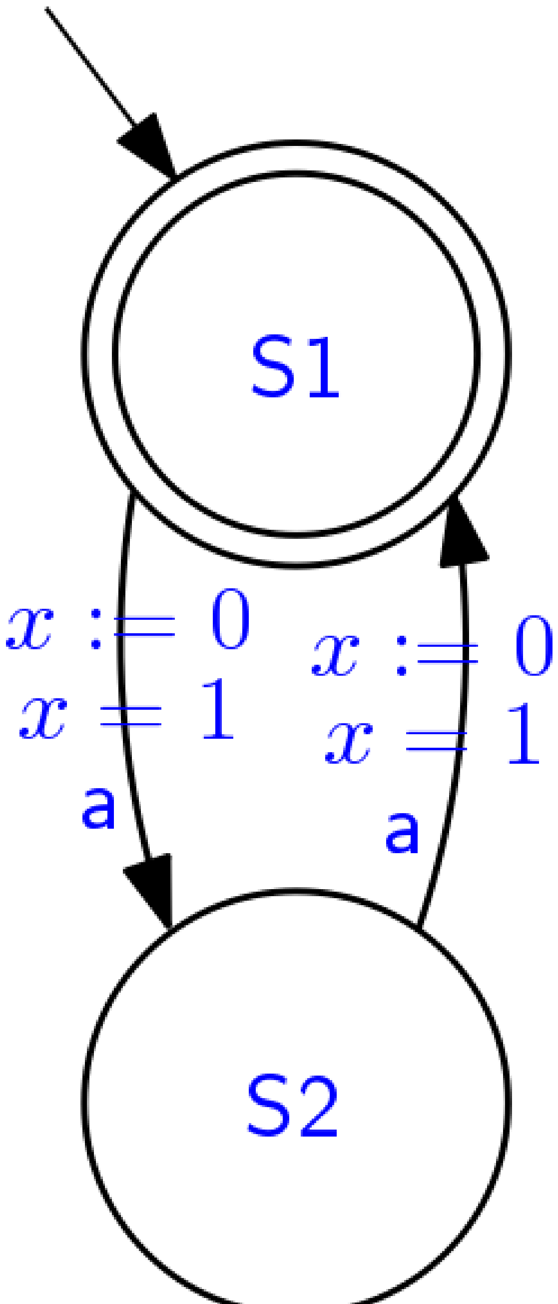
In all these exercises, we assume the nonnegative real numbers as time domain, unless explicitly stated otherwise.

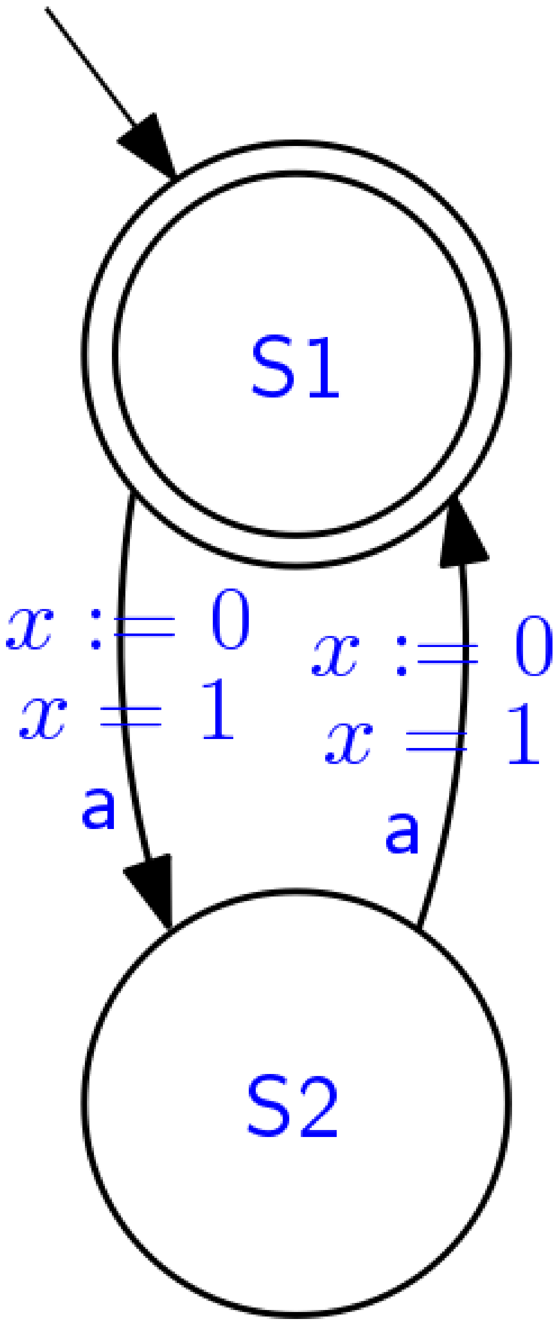
Exercises:
Does the property hold?

Does the property hold?



[] a

Does the property hold?

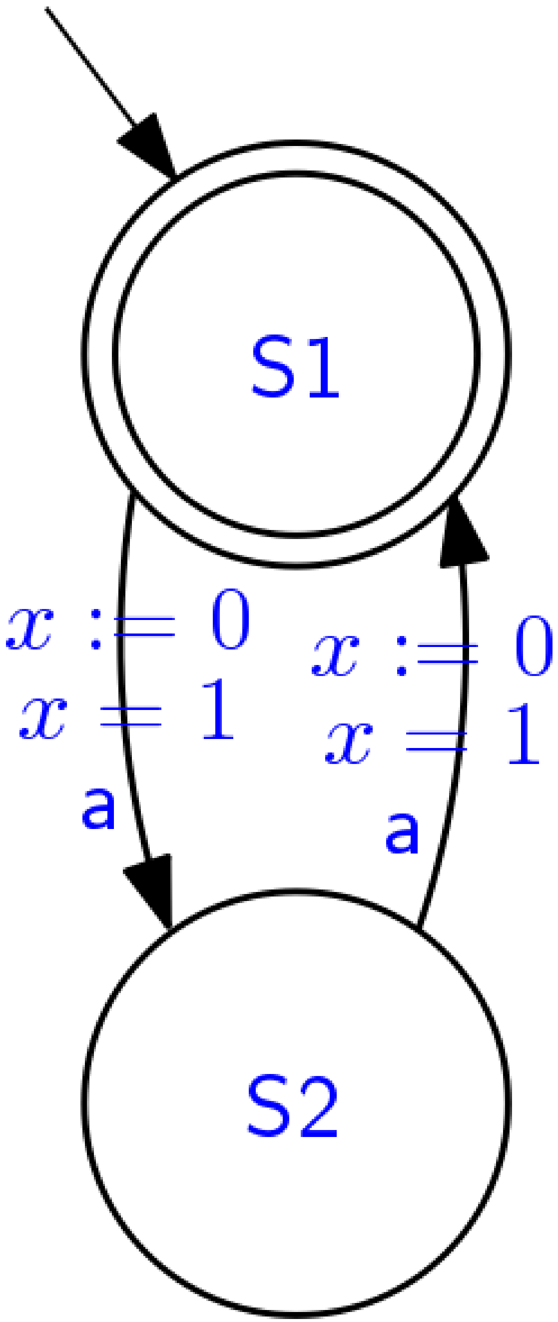


[] a

Yes:

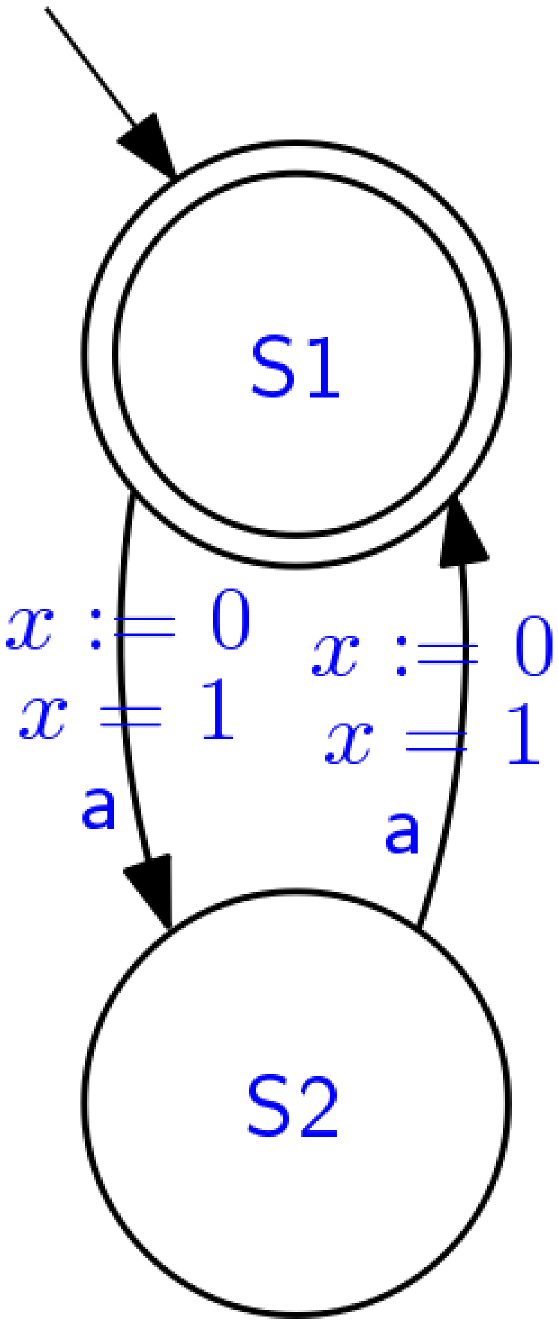
- it simply means that a holds at every position in the word (if any)

Does the property hold?



[] (<>=1 a)

Does the property hold?

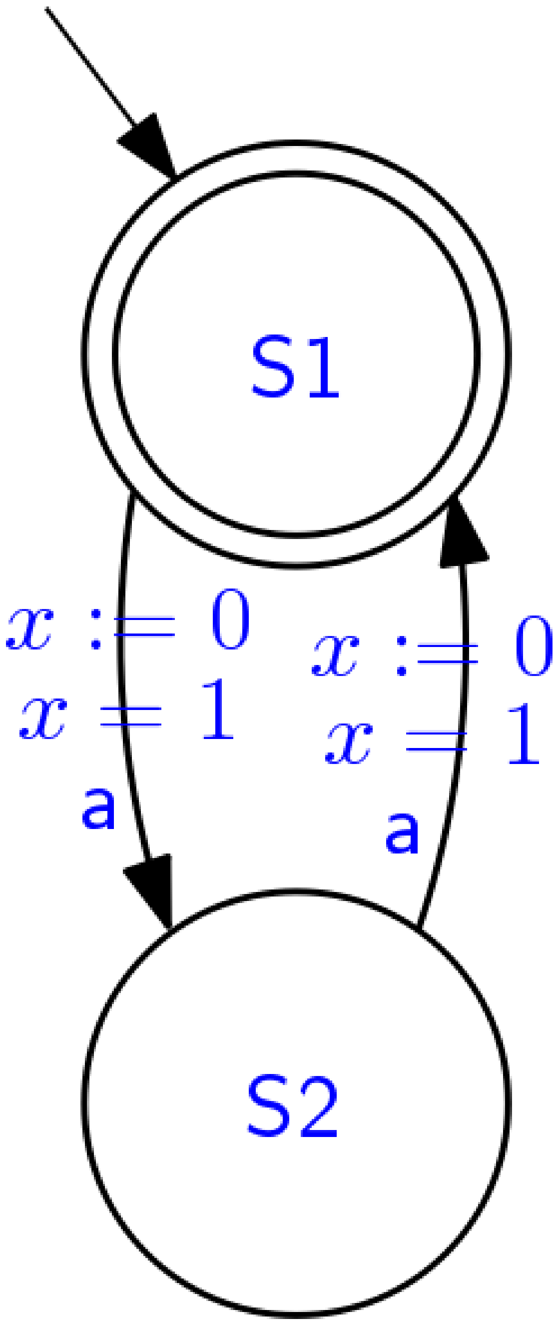


[] (<>=1 a)

No:

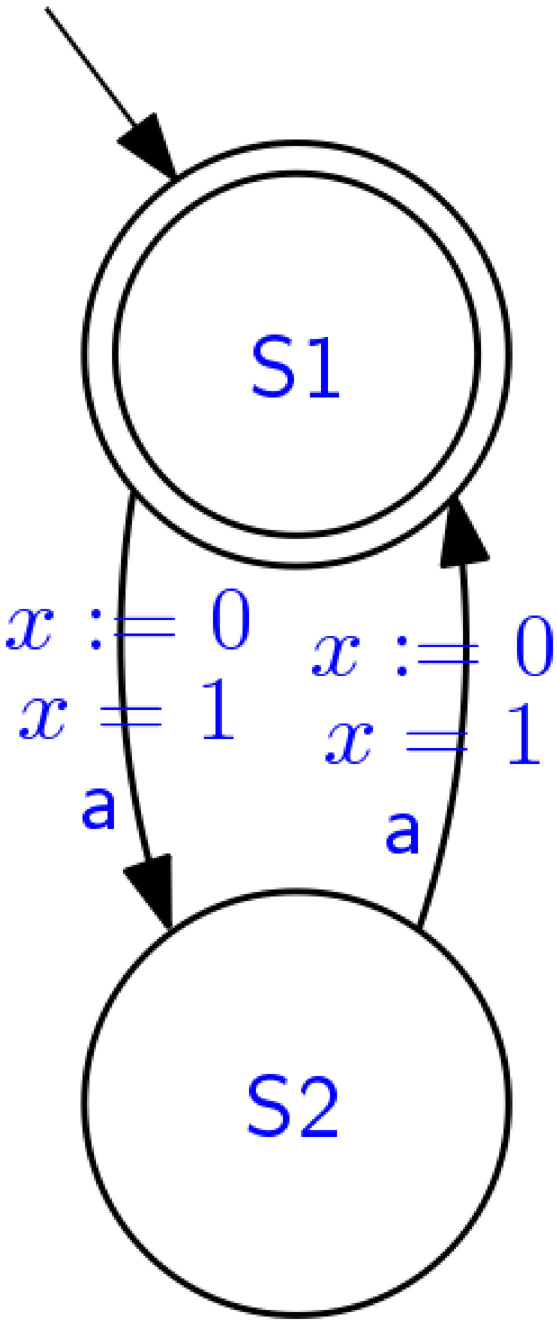
- this requires that there is always a future position, 1 time unit in the future, where a holds
- but this is not the case in the last position of any (non-empty) timed word

Does the property hold?



[] ([]=1 a)

Does the property hold?

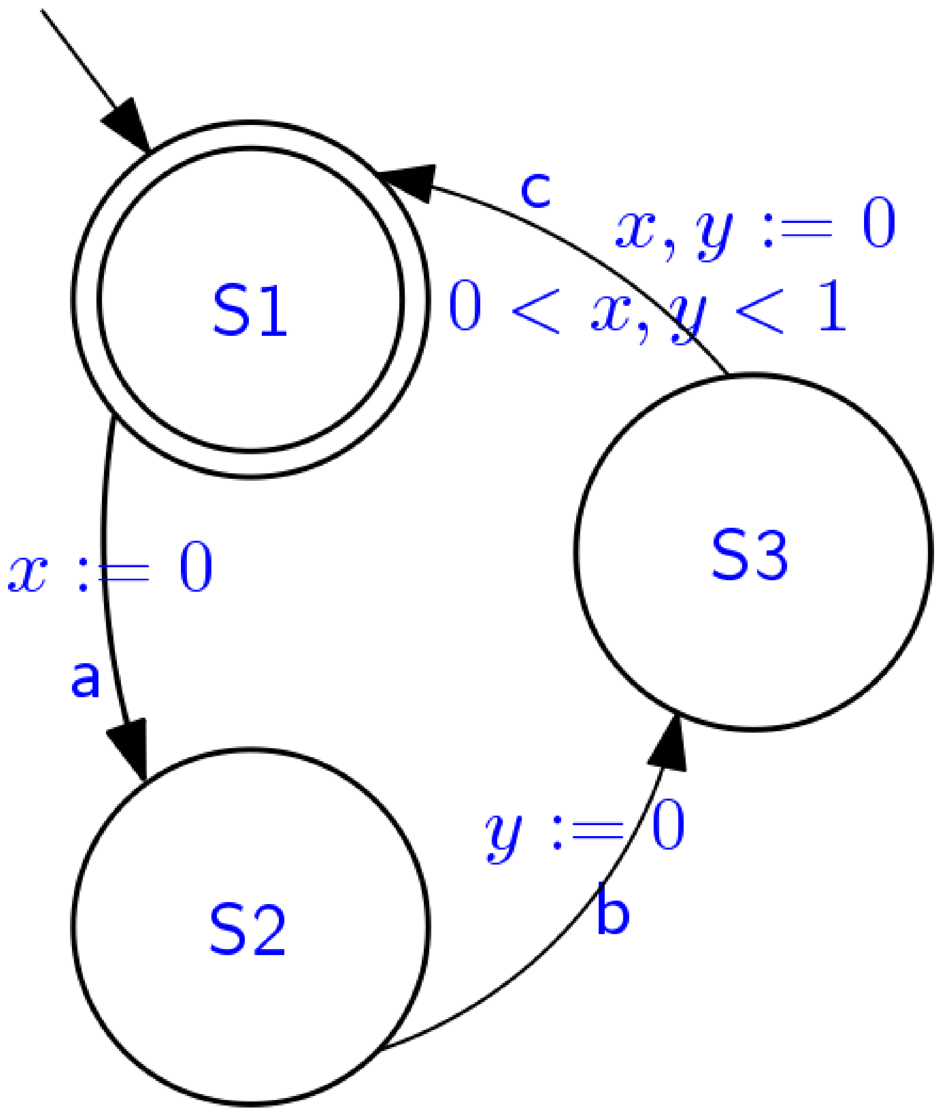


[] ([]=1 a)

Yes:

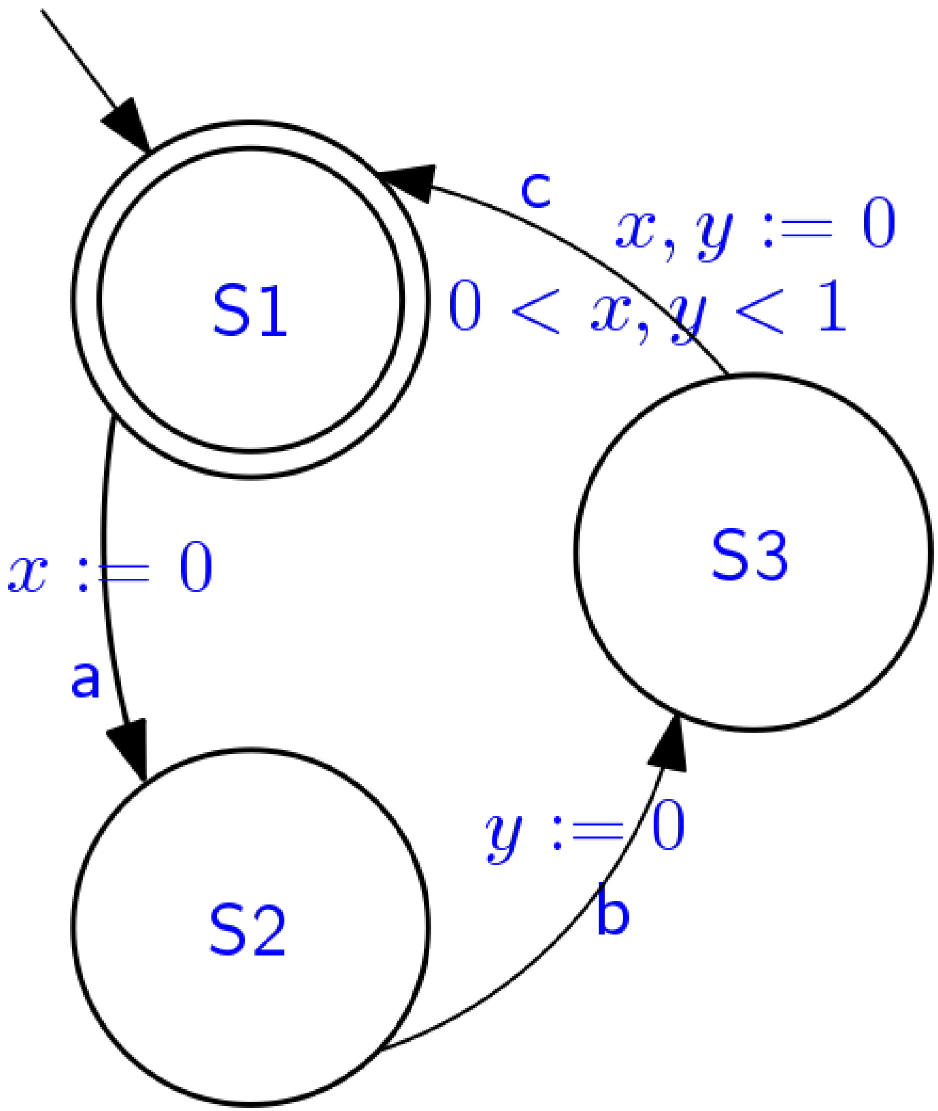
- the formula just requires that there if there is a future position 1 time unit in the future, then a holds there
- the automaton accepts only a's every time unit, hence the property is satisfied by any word accepted by the automaton

Does the property hold?



[] (a ⇒ <>(0,1) c)

Does the property hold?



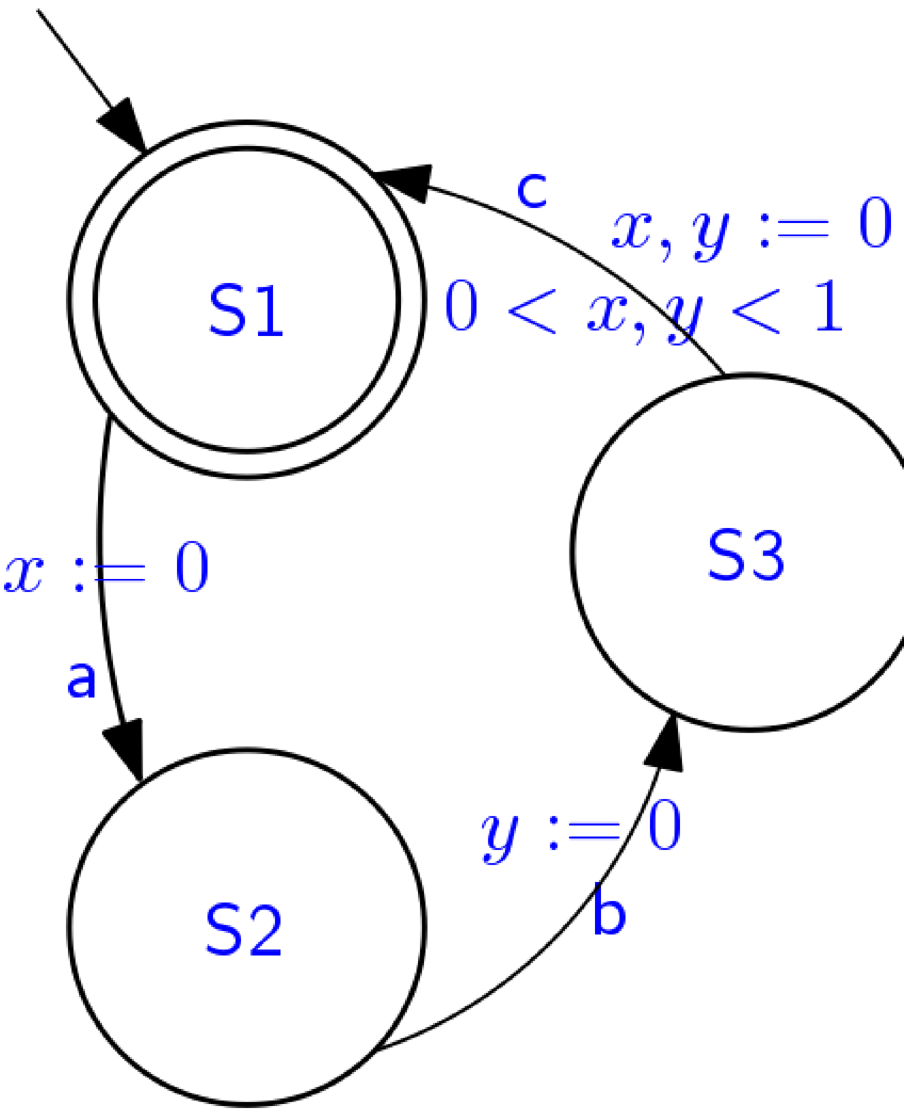
$[\,]\ (a \Rightarrow \langle\rangle_{(0,1)}\ c)$

Yes:

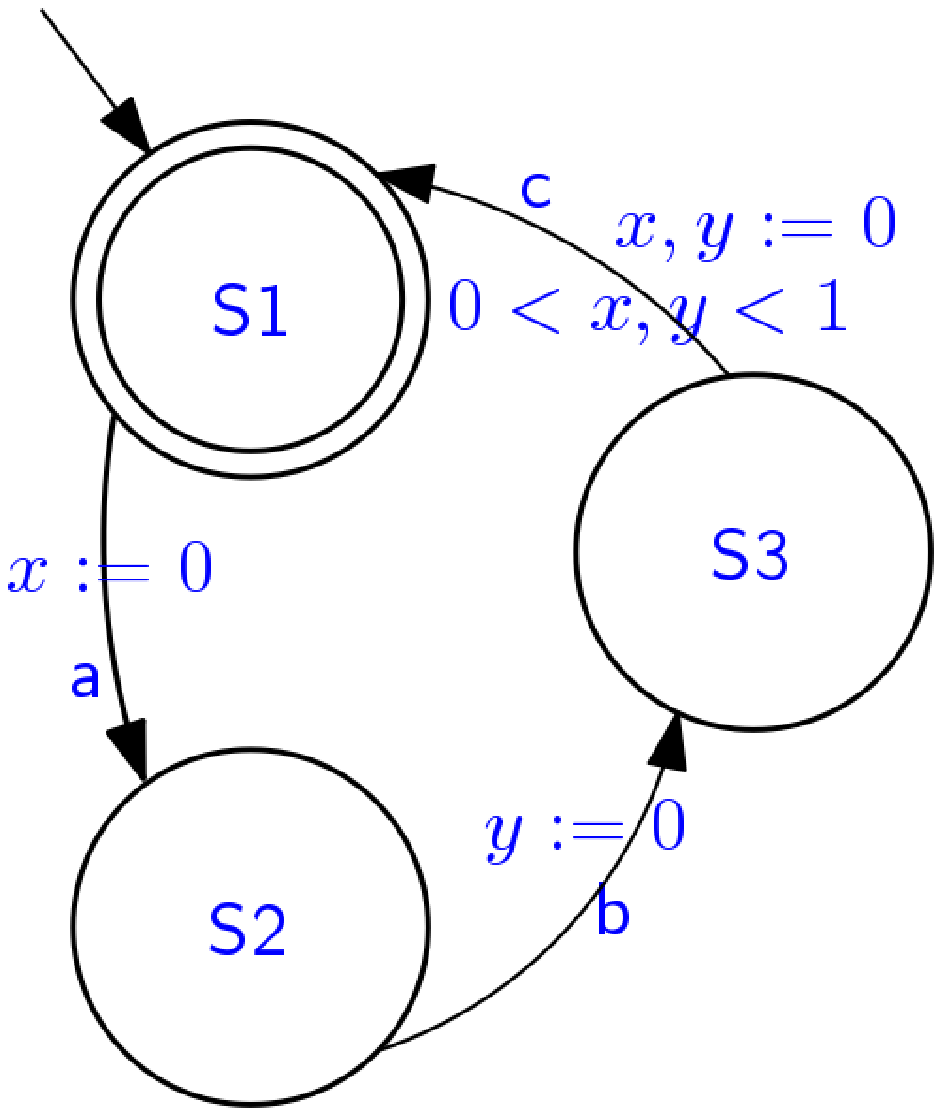
- clock x is reset upon reading a
- after that, it is checked upon reading c
- the constraint requires that x is in the range (0,1)

Does the property hold?

$[\,]\ (\ a \Rightarrow\ <>(0,1)\ b)$



Does the property hold?



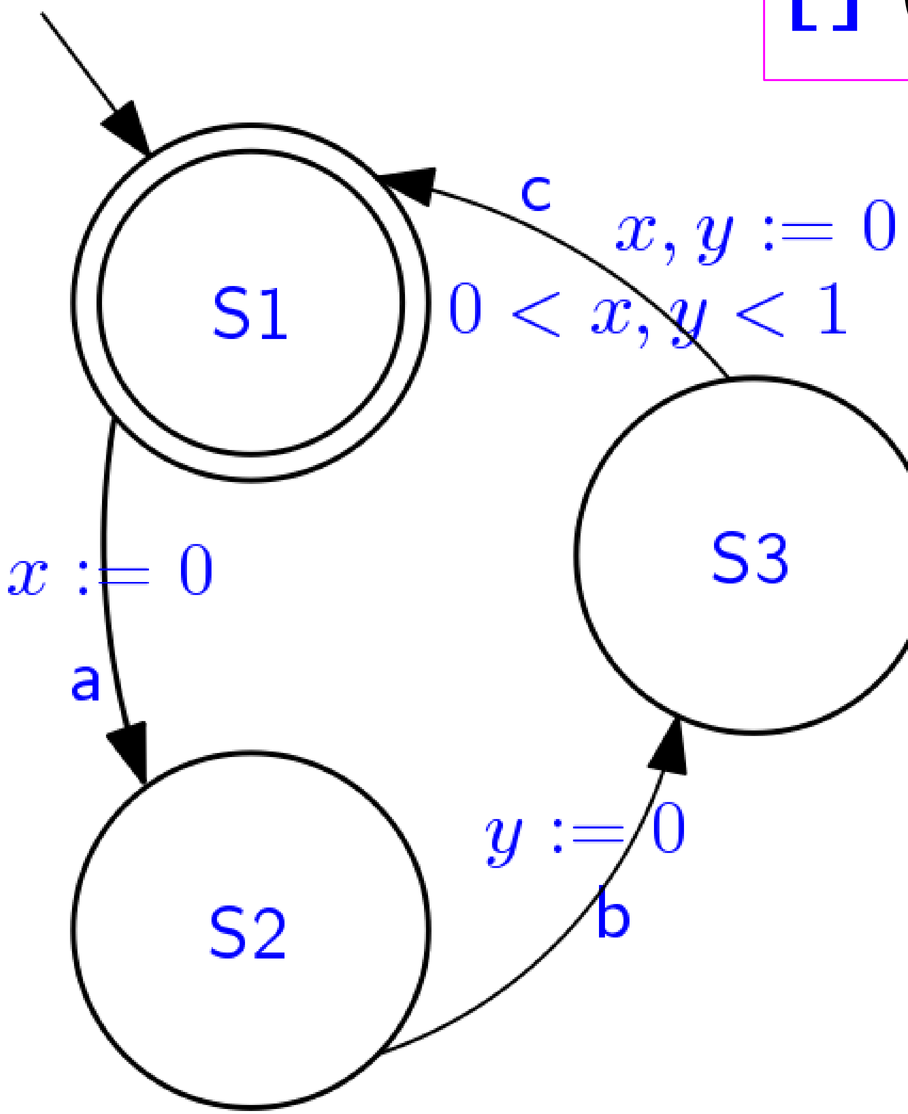
$[]\ (\ a \Rightarrow \langle\rangle(0,1)\ b)$

Yes:

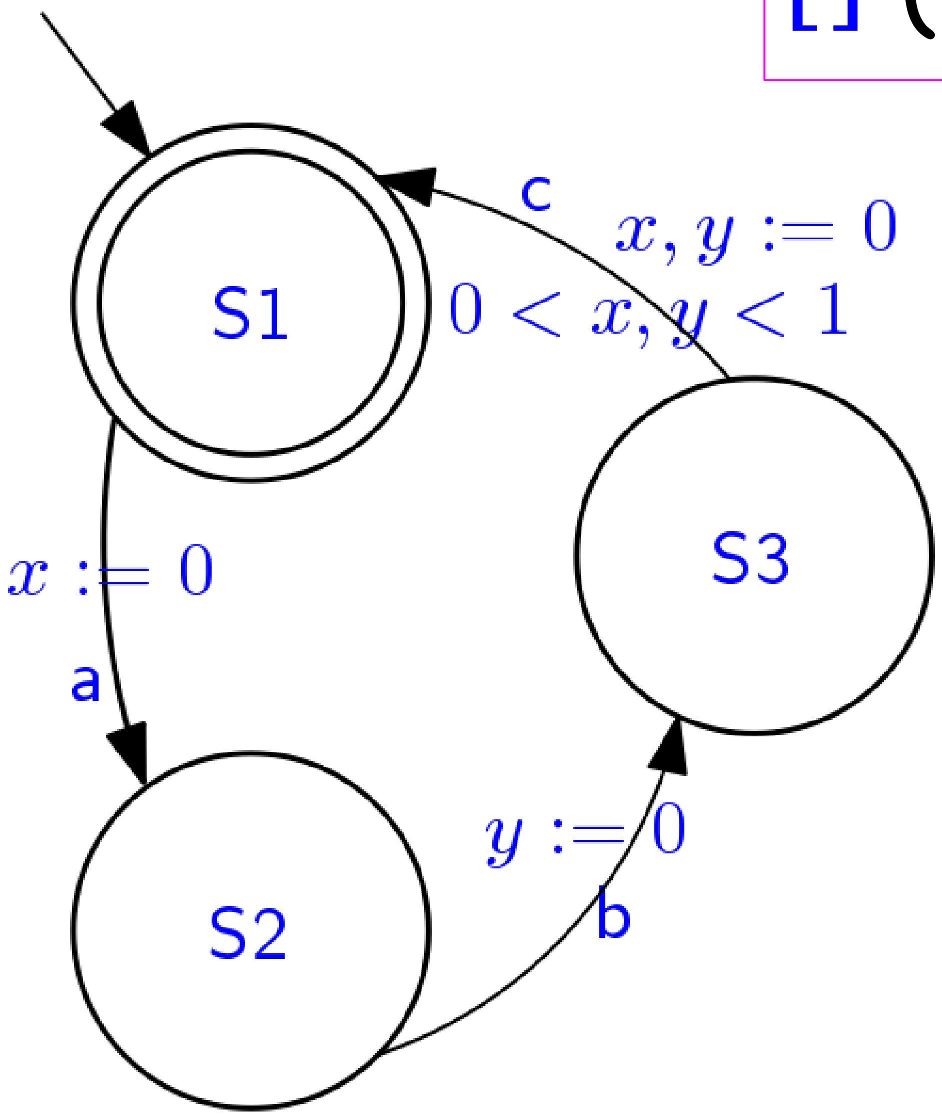
- clock x is reset upon reading a; after that, it is checked upon reading c, which is always preceded by a reading of b
- if b occurs later than or exactly after 1 time unit since the reading of b, the same occurs for the reading of c
- in this case the constraint on x would be violated

Does the property hold?

$[]\,(\,a \Rightarrow (a \vee b)\ U(0,1)\ c)$



Does the property hold?



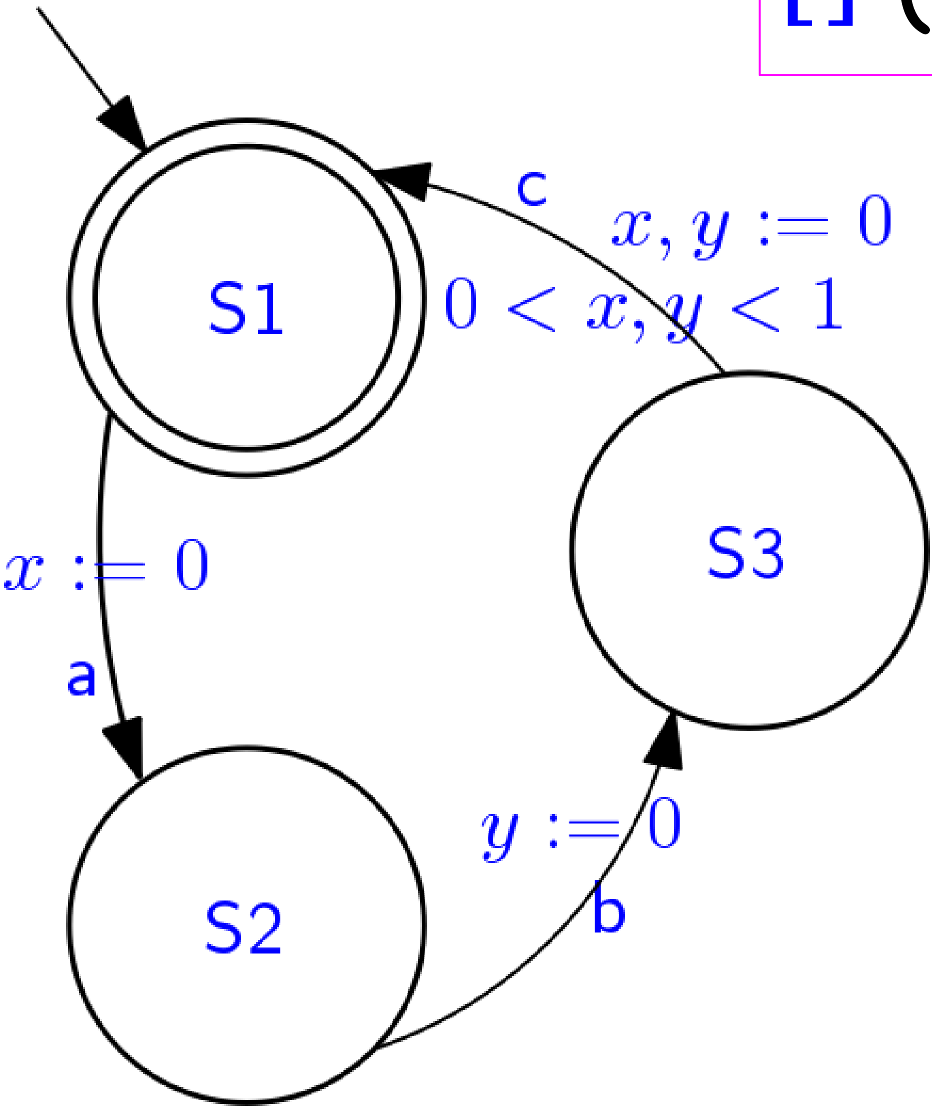
$[]\,(a \Rightarrow (a \vee b)\ U(0,1)\ c)$

Yes:

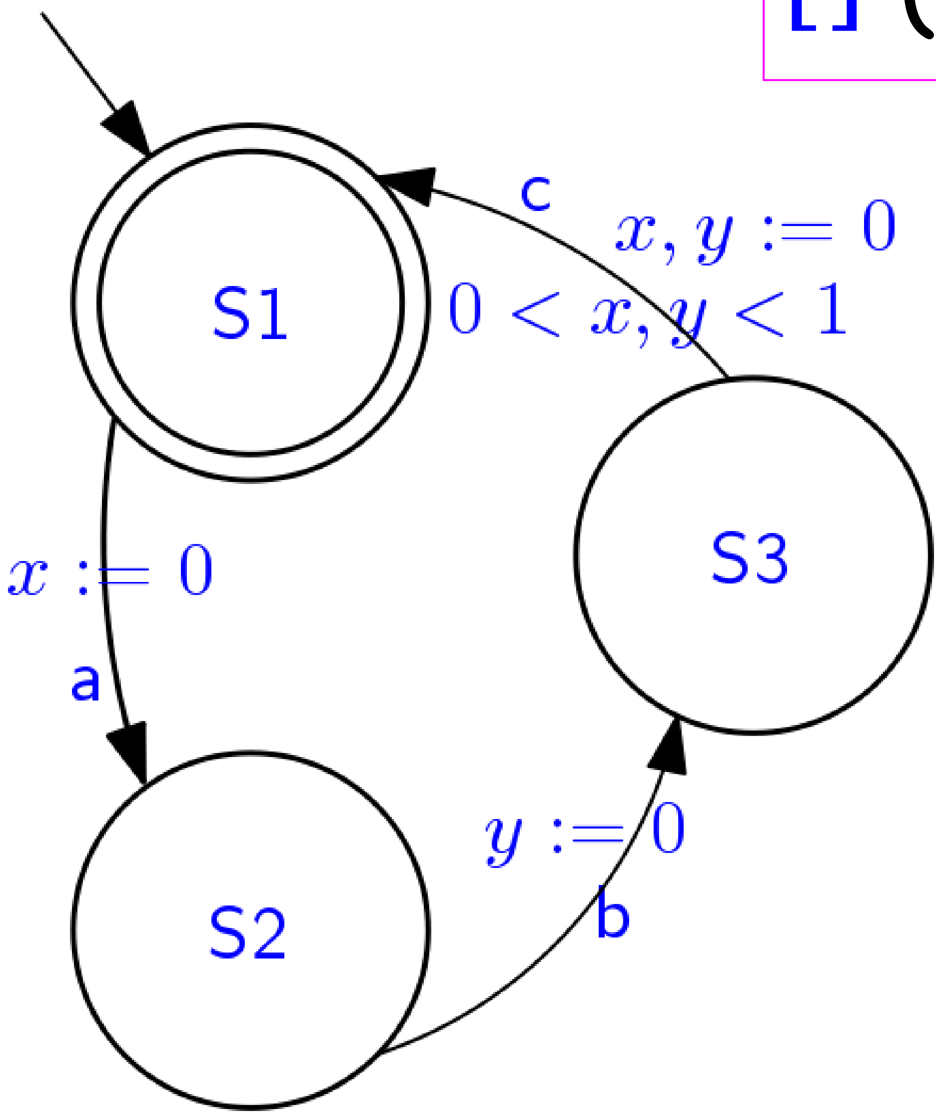
- clock x is reset upon reading a
- after that there is one reading of b followed by a reading of c, which satisfies the sequence of events required by the until formula
- as far as timing is concerned, c must occur within interval of time (0,1) since a occurred because of the clock constraint 0 < x,y < 1

Does the property hold?

$[]\,(\,a \Rightarrow (a \vee b)\; U(1,2)\; c)$



Does the property hold?



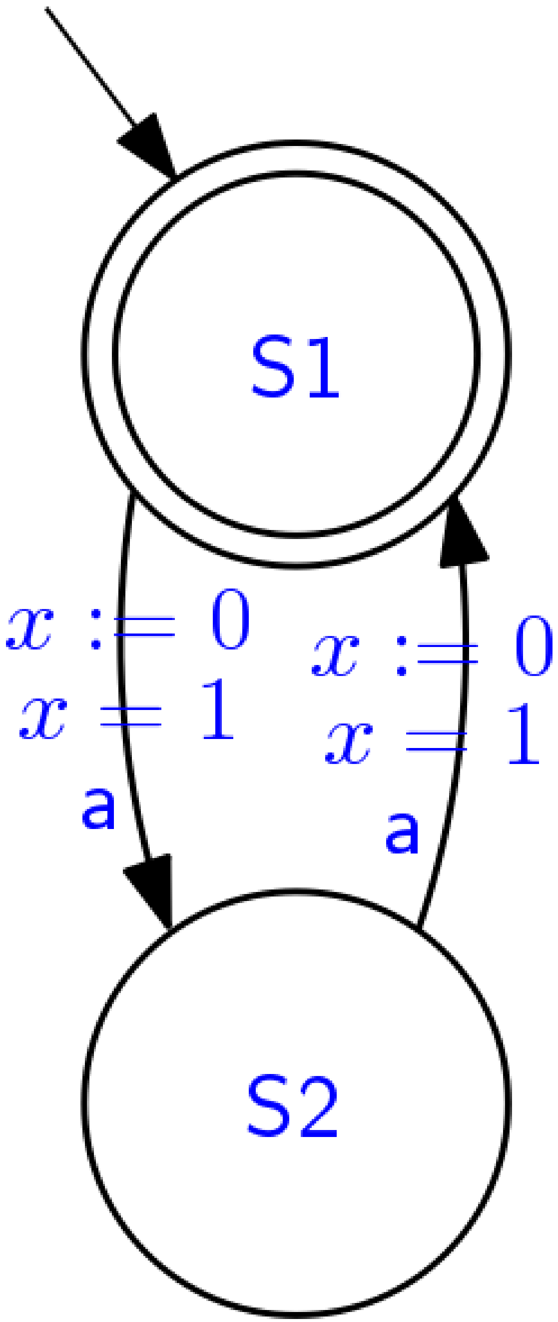
$[]$ (a ⇒ (a∨b) U(1,2) c)

No:

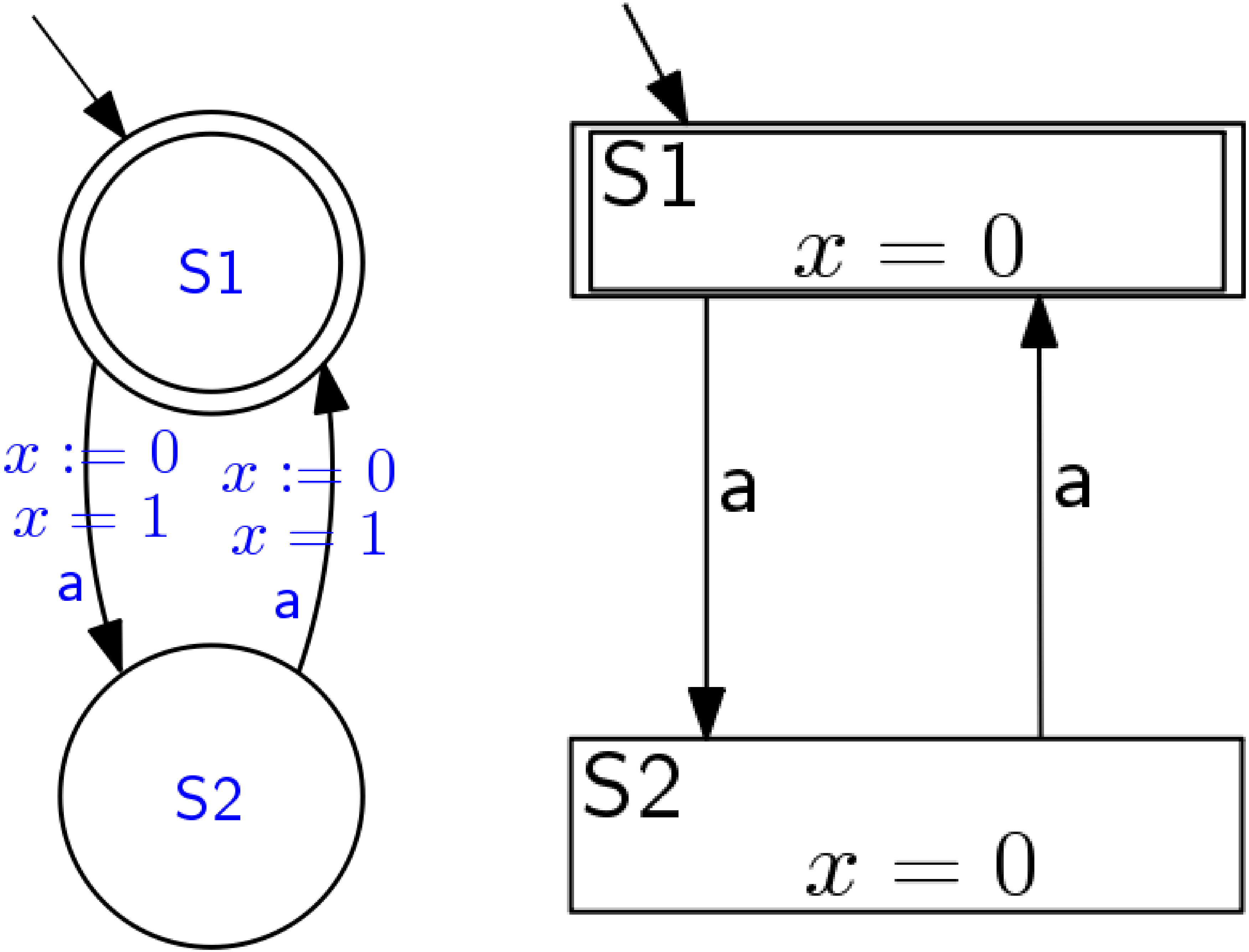
- if the "next" c is considered w.r.t when a occurs, it cannot happen in interval (1,2)
- if a successive occurrence of c is considered, it is preceded by at least another occurrence of c, which is not admitted by a∨b

Exercises:
Region automaton construction

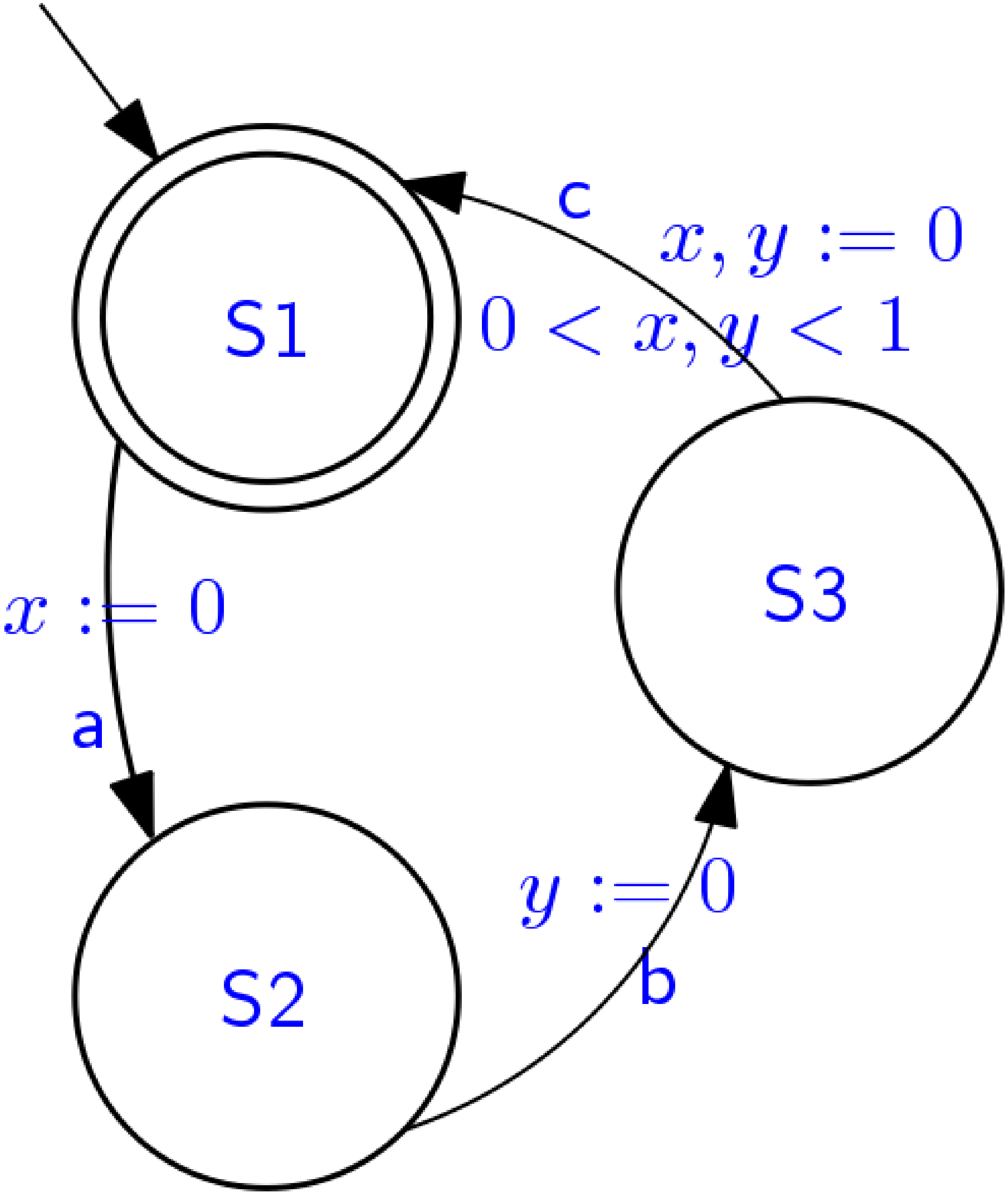
Build the region automaton for:



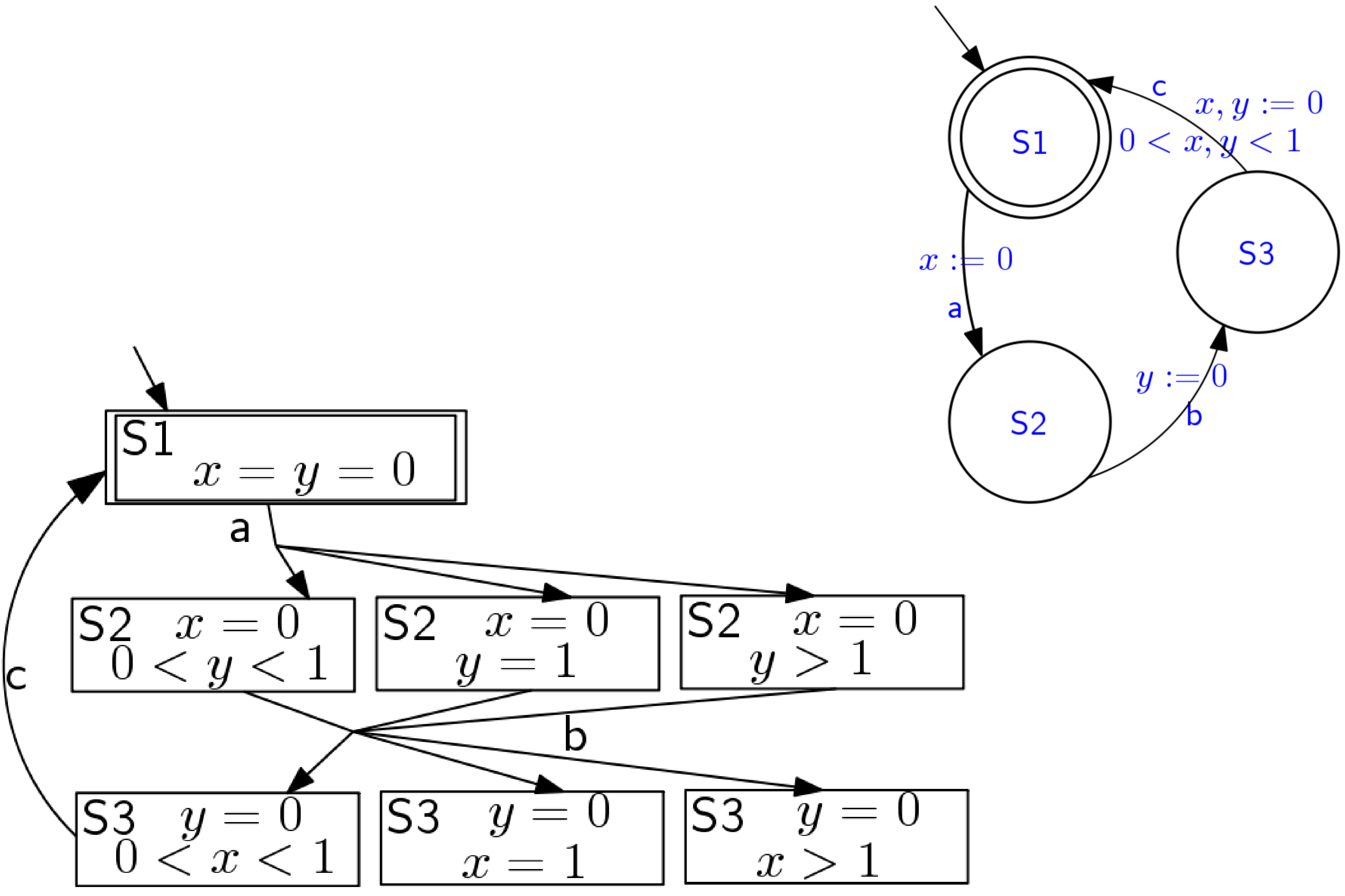
Build the region automaton for:



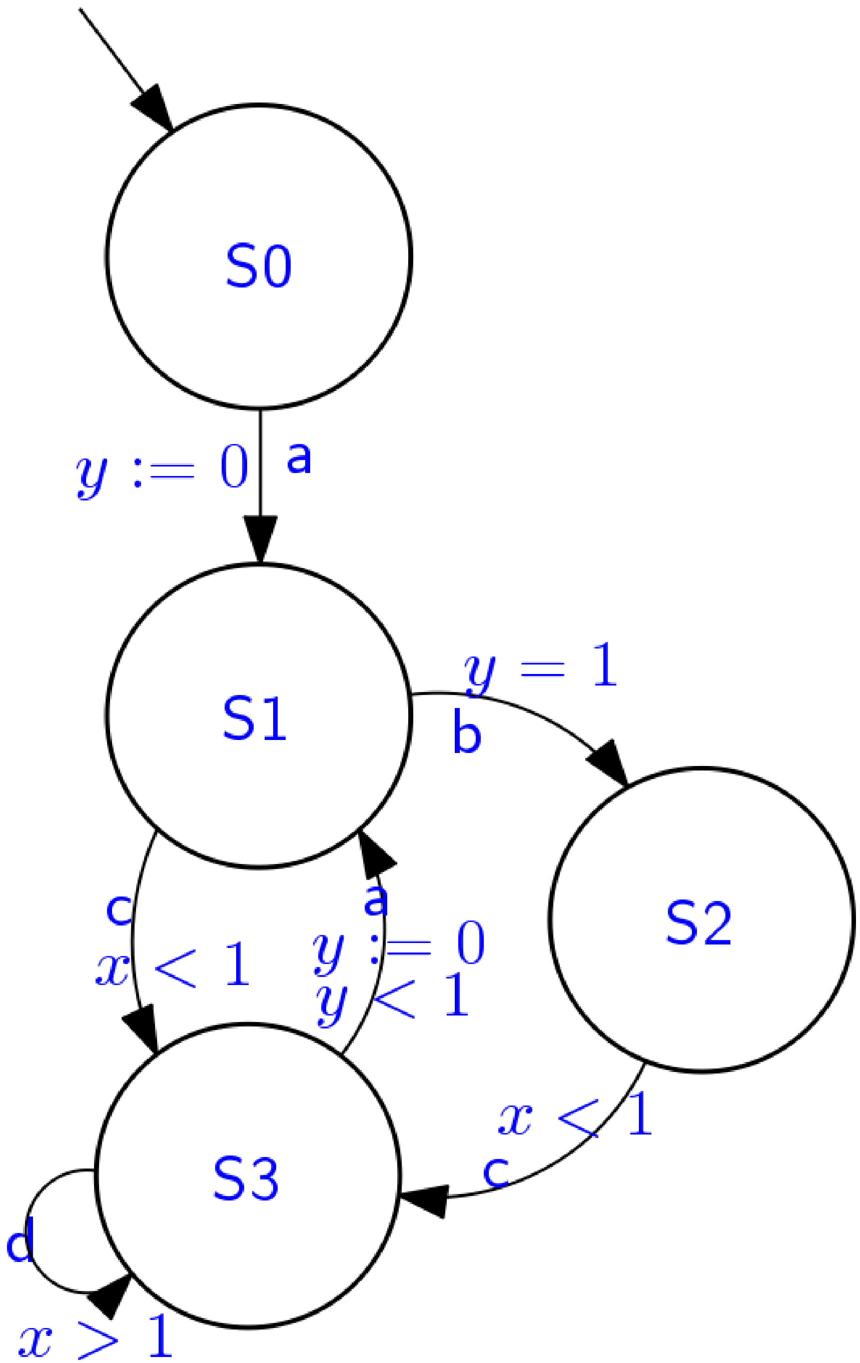
Build the region automaton for:



Build the region automaton for:

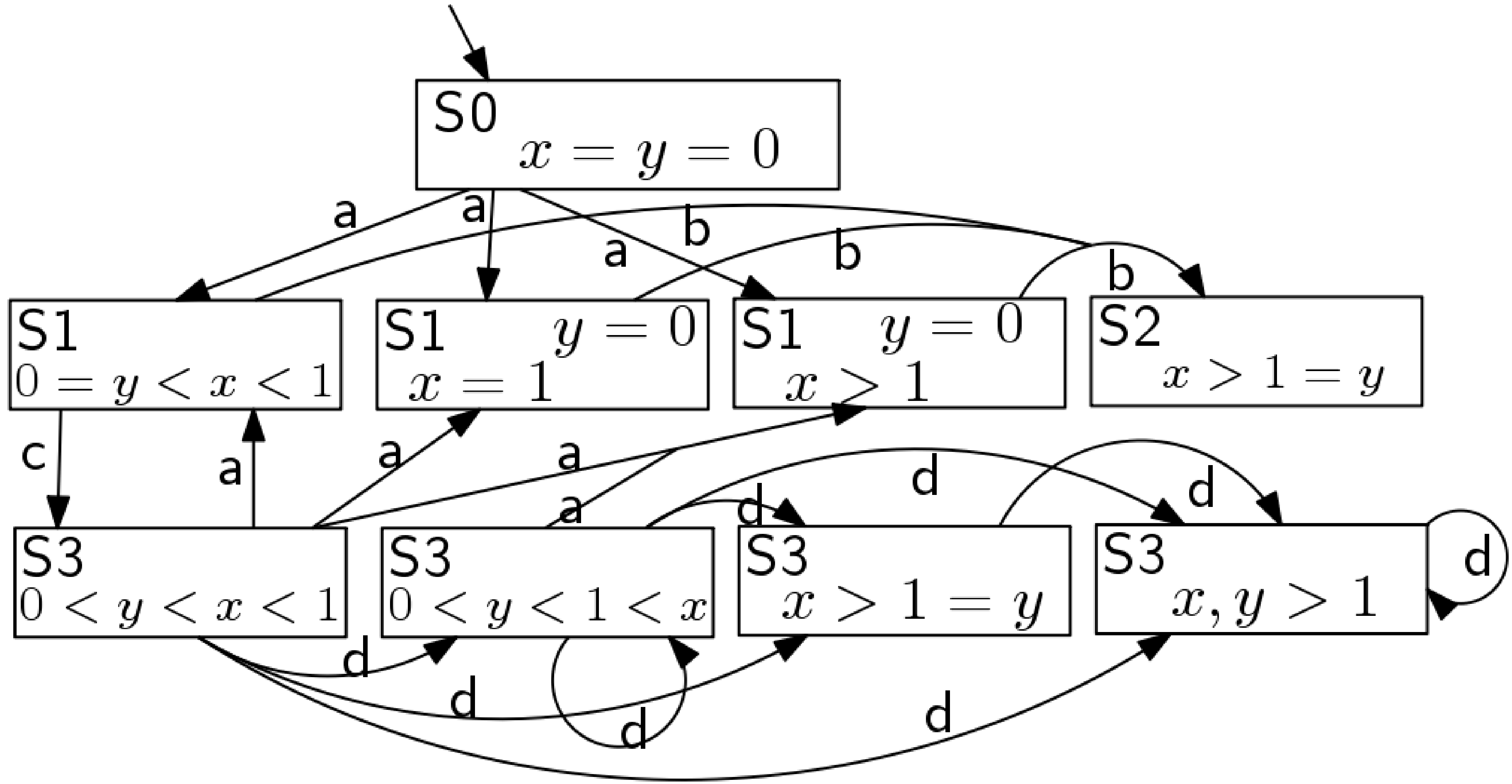


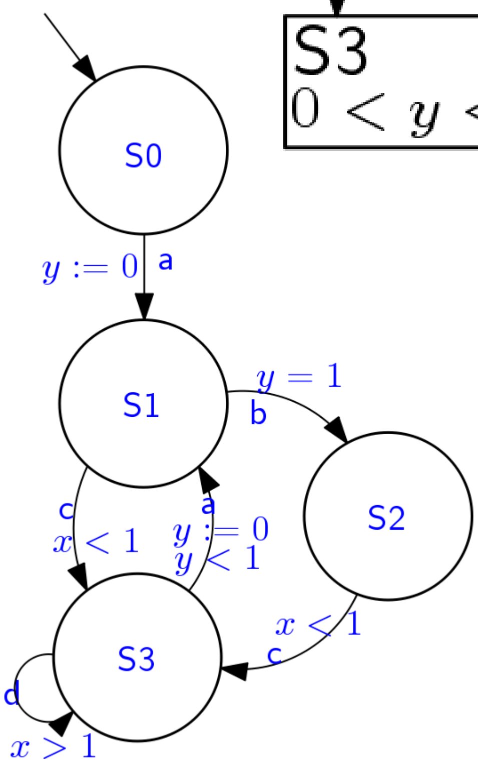
Build the region automaton for:



Example from: Alur & Dill, 1994

Build the region automaton for:





Example from: Alur & Dill, 1994

Exercises:
Semantics of derived operators

MTL derived operators: always

Prove that the satisfaction relation

$$w, i \models [\,]\langle a,b\rangle\, F$$

for bounded always, defined as:

$$[\,]\langle a,b\rangle\, F \triangleq \neg\,(\text{True } U\langle a,b\rangle\, \neg F)$$

is equivalent to:

for all $i \leq j \leq n$ such that $t(j) - t(i) \in \langle a,b\rangle$ it is: $w, j \models F$

MTL derived operators: always

w, i ⊨ []<a,b> F

iff

w, i ⊨ ¬ (True U<a,b> ¬F)   (definition of bounded always)

iff

it is not the case that:
for some i ≤ j ≤ n such that t(j) – t(i) ∈ <a,b> it is: w, j ⊨ ¬ F
and for all i ≤ k < j it is w, k ⊨ True

(definition of bounded until)

iff

for all i ≤ j ≤ n such that t(j) – t(i) ∈ <a,b> it is: not w, j ⊨ ¬ F
or for all i ≤ k < j it is w, k ⊨ False

(push negation inward)

iff

for all i ≤ j ≤ n such that t(j) – t(i) ∈ <a,b> it is: not w, j ⊨ ¬ F
(dropping false term in disjunction)

iff

for all i ≤ j ≤ n such that t(j) – t(i) ∈ <a,b> it is: w, j ⊨ F
(simplification of double negation)

MTL derived operators: X and X-

Compare the semantics of:

$$X+\ F \triangleq \mathit{True}\ U_{=1}\ F$$

with the semantics of:

$$X-\ F \triangleq F\ U_{>0}\ \mathit{True}$$

Semantic of X+

w, i ⊨ X+ F

iff

w, i ⊨ True U=1 F (definition of X+)

iff

for some i ≤ j ≤ n such that t(j) - t(i) = 1 it is: w, j ⊨ F

and for all i ≤ k < j it is w, k ⊨ True

(definition of bounded until)

iff

for some i ≤ j ≤ n such that t(j) = t(i) + 1 it is: w, j ⊨ F

(simplify term)

Semantic of X-

w, i ⊨ X- F

iff

w, i ⊨ F $U_{>0}$ True (definition of X-)

iff

for some i ≤ j ≤ n such that t(j) – t(i) > 0 it is: w, j ⊨ True
and for all i ≤ k < j it is w, k ⊨ F

(definition of bounded until)

iff

for some i < j ≤ n it is: w, j ⊨ True and for all i ≤ k < j it is w, k ⊨ F
(timestamps are strictly increasing by assumption)

iff

i < n and w, i ⊨ F
(take j = i+1 so that [i, j) = [i,i])

Exercises:
Equivalence of MTL formulas

Comparison of formulas

Is formula:

[] <>>0 True

satisfied by any timed word?

Is formula satisfied?

Semantics of: $w \models [\,] <>>0$ True

for all positions $1 \leq i \leq n$: $w,i \models <>>0$ True

Semantics of: $w,i \models <>>0$ True

for some $j > i$ it is: $w,j \models$ True

i.e.: $i < n$

Hence: $w \models [\,] <>>0$ True

holds only for the empty word!

Comparison of formulas

Is formula:

[] <>≥0 True

satisfied by any (non-empty) timed word?

Is formula satisfied?

Semantics of: $w \vDash [\,]\,\langle\rangle_{\geq 0}$ True

for all positions $1 \leq i \leq n$: $w,i \vDash \langle\rangle_{\geq 0}$ True

Semantics of: $w,i \vDash \langle\rangle_{\geq 0}$ True

for some $j \geq i$ it is: $w,j \vDash$ True

i.e.: True

because one can always take $j = i$

Hence: $w \vDash [\,]\,\langle\rangle_{\geq 0}$ True

holds for any word.

Comparison of formulas

Is formula:

<>[a,b] <>[c,d] q

equivalent or non-equivalent to:

<>[a+c,b+d] q

Inequivalent formulas

Informal meaning of: <>[a,b] <>[c,d] q

- let i be the current position
- there exist a future position j > i in the word with time in [a,b] relative to i such that:
- there exist another future position k > j in the word with time in [c,d] relative to j, where q holds
- in all, the time at which q holds is in [a+c, b+d] relative to i

Informal meaning of: <>[a+c,b+d] q

- let i be the current position
- there exist another future position k > i in the word with time in [a+c,b+d] relative to i, where q holds

Hence, for instance: timed word w = ({}, 3) ({q}, 3+b+c)

is such that: w satisfies <>[a+c,b+d] q but it does not satisfy <>[a,b] <>[c,d] q because there is no intermediate position between the first and the one where q holds