



Einführung in die Programmierung Introduction to Programming

Prof. Dr. Bertrand Meyer

Exercise Session 9



- Feedback on the mock exam

- Recursion
 - Recursion
 - Recursion
 - Recursion
 - Recursion

- Basic data structures
 - Arrays
 - Linked Lists
 - Hashtables

Recursion: an example



- Fibonacci sequence:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

- How can we calculate the n-th Fibonacci number?

- Recursive formula:

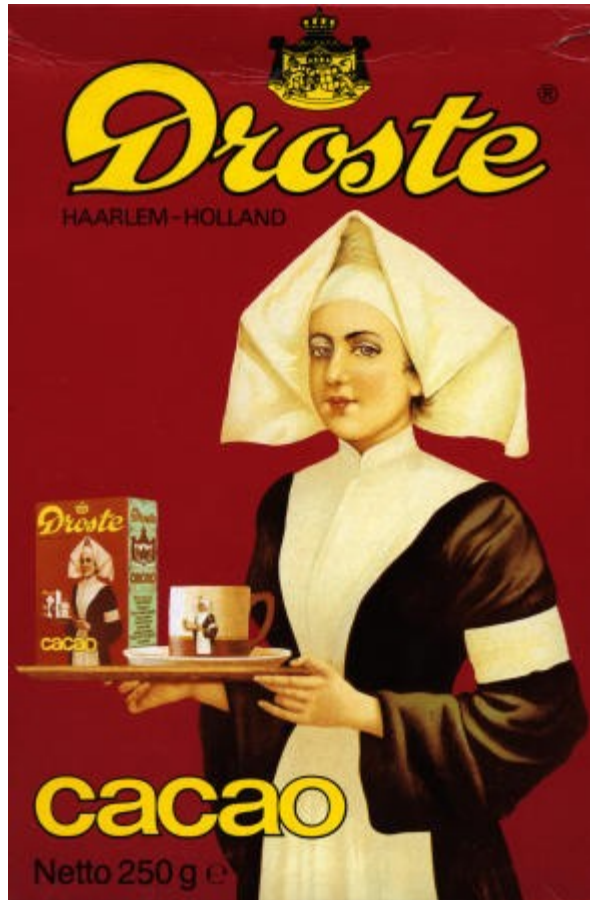
$$F(n) = F(n-1) + F(n-2) \text{ for } n > 1$$

$$\text{with } F(0) = 0, F(1) = 1$$

Recursion: a second example



- Another example of recursion



Source: en.wikipedia.org/wiki/Recursion

<https://www.flickr.com/photos/tin-g>

A recursive feature



```
fibonacci(n: INTEGER): INTEGER
```

```
do
```

```
  if n = 0 then
```

```
    Result := 0
```

```
  elseif n = 1 then
```

```
    Result := 1
```

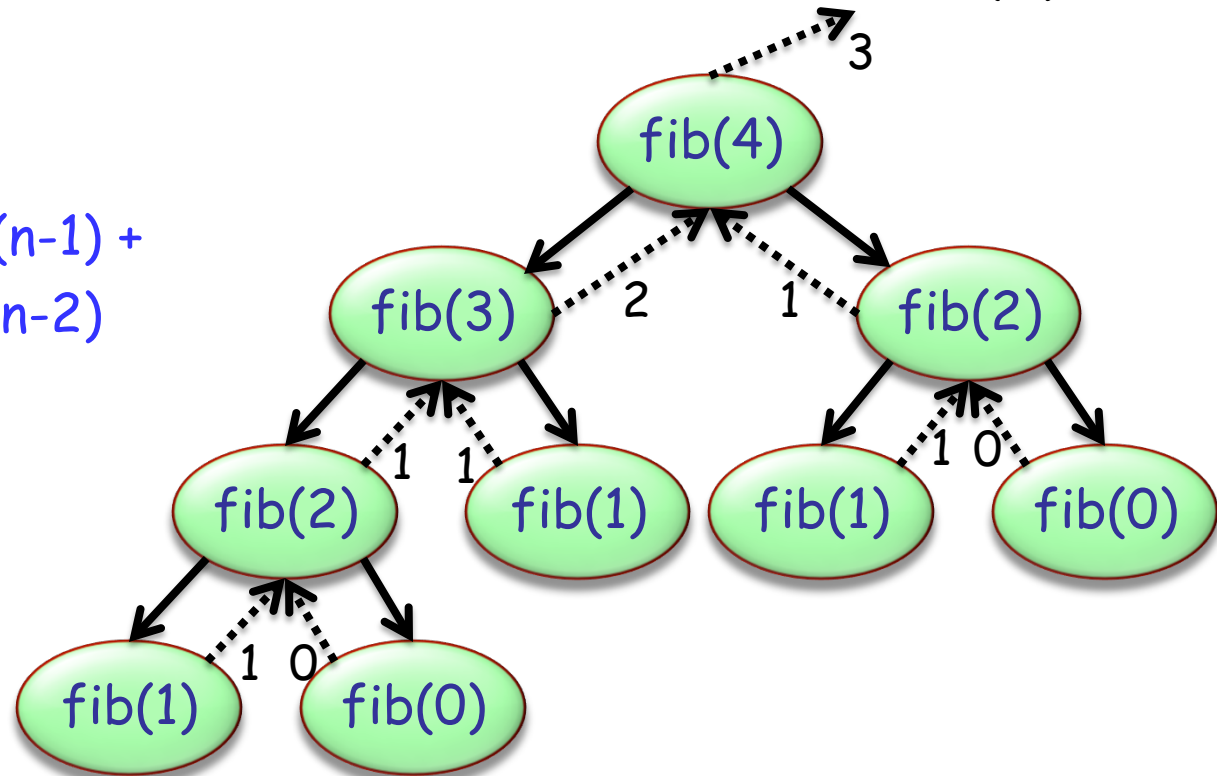
```
  else
```

```
    Result := fibonacci(n-1) +  
             fibonacci(n-2)
```

```
  end
```

```
end
```

➤ Calculate fibonacci(4)



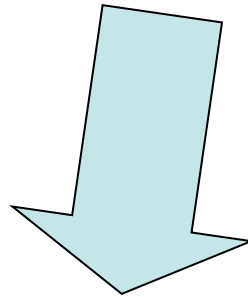


A definition for a concept is **recursive** if it involves an instance of the concept itself

- The definition may use "*instances of concept itself* "
- *Recursion* is the use of a recursive definition

„To iterate is human, to recurse - divine!“

but ... computers are built by humans 

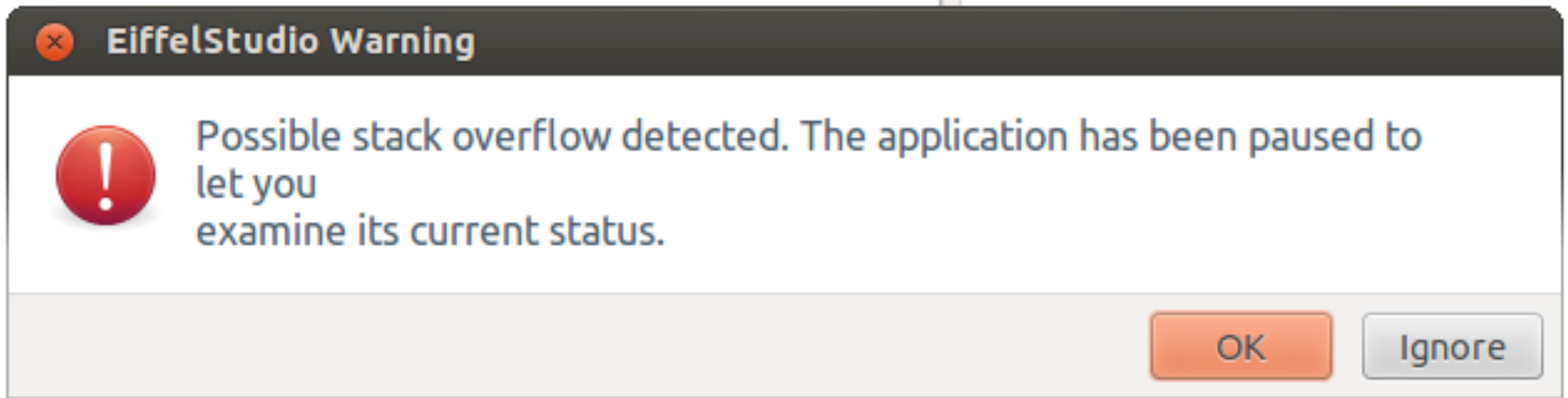


Better use iterative approach if reasonable? 



- Every recursion could be rewritten as an iteration and vice versa.
- Recursion is slower because all functions calls must be stored in memory to allow the return back to the caller functions.
- It's more intuitive in cases where it mimics our approach to the problem, e.g. generating Fibonacci numbers.
- Data structures such as trees are easier to explore with recursion.

Be careful when using recursion!



- Stack: a region of memory that store temporary data created by your program.

Exercise: Printing numbers

Hands-On

- If we pass $n = 4$, what will be printed?

```
print_int (n: INTEGER)  
do  
  print (n)  
  if  $n > 1$  then  
    print_int (n - 1)  
  end  
end
```

4321

```
print_int (n: INTEGER)  
do  
  if  $n > 1$  then  
    print_int (n - 1)  
  end  
  print (n)  
end
```

1234

Exercise: Reverse string

Hands-On

- Print a given string in reverse order using a recursive function.

Exercise: Solution



```
class APPLICATION

  create
    make

  feature
    make
      local
        s: STRING
      do
        create s.make_from_string ("poldomangia")
        invert(s)
      end

    invert (s: STRING)
      require
        s /= Void
      do
        if not s.is_empty then
          invert (s.substring (2, s.count))
          print (s[1])
        end
      end
    end
  end
end
```

Exercise: Sequences

Hands-On

- Write a recursive and an iterative program to print the following:

111,112,113,121,122,123,131,132,133,
211,212,213,221,222,223,231,232,233,
311,312,313,321,322,323,331,332,333,

- Note that the recursive solution can use loops too.

Exercise: Recursive solution



cells: ARRAY [INTEGER]

handle_cell (n: INTEGER)

local

i: INTEGER

do

from

i := 1

until

i > 3

loop

cells [n] := i

if (n < 3) then

handle_cell (n+1)

else

print (cells [1].out+cells [2].out+cells [3].out+",")

end

i := i + 1

end

end

Exercise: Iterative solution



```
from
  i := 1
until
  i > 3
loop
  from
    j := 1
  until
    j > 3
  loop
    from
      k := 1
    until
      k > 3
    loop
      print (i.out+j.out+k.out+",")
      k := k + 1
    end
    j := j + 1
  end
  i := i + 1
end
```



An array is a very fundamental data-structure, which is very close to how your computer organizes its memory. An array is characterized by:

- Constant time for random reads/writes
- Costly to resize (including inserting elements in the middle of the array)
- Must be indexed by an integer
- Generally very space efficient.

In Eiffel the basic array class is generic, `V_ARRAY [G]`.

Using Arrays

Which of the following lines are valid?
Which can fail, and why?

➤ `my_array : V_ARRAY [STRING]`

Valid, can't fail

➤ `my_array ["Fred"] := "Sam"`

Invalid

➤ `my_array [10] + "'s Hat"`

Valid, can fail

➤ `my_array [5] := "Ed"`

Valid, can fail

➤ `my_array.force ("Constantine", 9)`

Valid, can't fail

Which is not a constant-time array operation?



- Linked lists are one of the simplest data-structures
- They consist of linkable cells

```
class LINKABLE[G]
```

```
  create
```

```
    set_value
```

```
  feature
```

```
    set_value(v: G)
```

```
      do
```

```
        value := v
```

```
      end
```

```
  value: G
```

```
  set_next(n: LINKABLE[G])
```

```
    do
```

```
      next := n
```

```
    end
```

```
  next: LINKABLE[G]
```

```
end
```

Suppose you keep a reference to only the head of the linked list, what is the running time (using big O notation) to:

- Insert at the beginning
- Insert in the middle
- Insert at the end
- Find the length of the list

$O(1)$

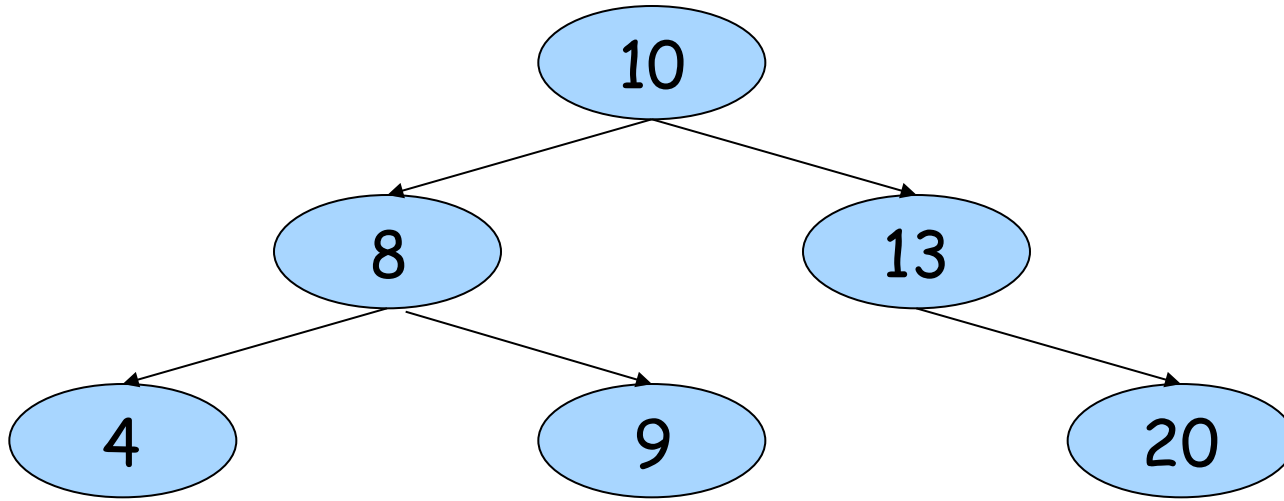
$O(n)$

$O(n)$

$O(n)$

What simple optimization could be made to make end-access faster?

Binary search tree



- A binary search tree is a binary tree where each node has a *COMPARABLE* value.
- Left sub-tree of a node contains only values less than the node's value.
- Right sub-tree of a node contains only values greater than or equal to the node's value.

Exercise: Adding nodes

Hands-On

- Implement command *put* (*n: INTEGER*) in class *NODE* which creates a new *NODE* object at the correct place in the binary search tree rooted by *Current*.
- Test your code with a class *APPLICATION* which builds a binary search tree using *put* and prints out the values using the traversal feature.
- Hint: You might need to adapt the traversal feature such that the values are printed out in order.

Exercise: Solution



- See code in IDE.

Exercise: Searching

Hands-On

- Implement feature *has* (*n*: *INTEGER*): *BOOLEAN* in class *NODE* which returns true if and only if *n* is in the tree rooted by *Current*.
- Test your code with a class *APPLICATION* which builds a binary search tree and calls *has*.

Exercise: Solution



➤ See code in IDE.