Alloy as a refactoring checker?

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Motivation

"As a program is evolved its complexity increases unless work is done to maintain or reduce it." *M. M. Lehman*

Motivation

- *Refactorings* are systematic changes to improve the structure of a program, e.g.
  - Simplify operations
  - Improve reusability
  - Increase readability
- Used for programs but also models or specifications
- Important: refactorings must preserve the external observable behavior

Motivation

- How to check *behavior-preservation*?
  - Usual approach: testing
  - Use template pairs (describing before and after state)
  - Use an automatic verification tool
- Subject of our work
  - *Can the Alloy Analyzer be used to verify behavior-preservation of refactorings for Z specifications?*

Overview

1. Translating a Z specification into the Alloy language
2. Defining *behavior-preservation* for refactorings in Z
3. Applying the Alloy Analyzer for verification

What is Alloy?

- Alloy = Alloy language + Alloy Analyzer
- developed by the Software Design Group at MIT
- Alloy language
  - Declarative specification language (based on first order logic)
  - Strongly inspired by Z
- Alloy Analyzer
  - SAT based constraint solver
  - Automatic simulation and analysis of Alloy models
  - A model **finder**: tries to find a model for a formula

Example of a translation

```
sig ELEMENT {}

sig Set {
  elements: set ELEMENT
}

pred Add_Elem[s, s': Set,
              e_in: ELEMENT]{
  e_in not in s.elements
  s'.elements = s.elements + e_in
}

/* run a simulation */
run {} for 3
```

$[ELEMENT]$

Set

$elements : \mathbb{P}\, ELEMENT$

Add_Elem

ΔSet

$e? : ELEMENT$

$e? \notin Set$

$elements' = elements \cup \{e?\}$

Structure of an Alloy model

```
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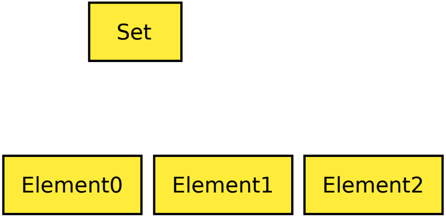
```

- Signatures define the state space
- Model consists of *atoms* and *relations*



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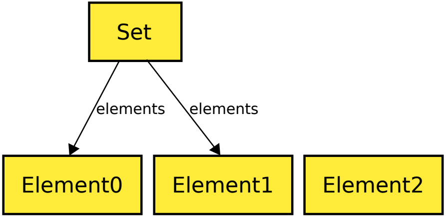

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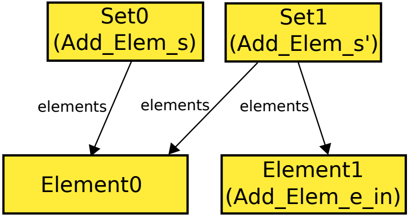
/* run a simulation */
run Add_Elem for 3
```

- Z operations are translated to predicates



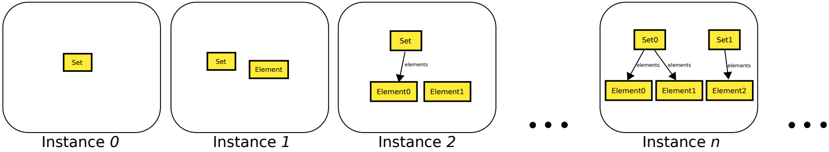
Checking properties of an Alloy model

- Use *assertions* to check properties of a model, e.g.

```
/* Assertion: there are no empty sets */
assert EmptySet { all s: Set | #s.elements > 0 }

check EmptySet for 3 but 2 Set
```

- Alloy Analyzer examines every possible instance



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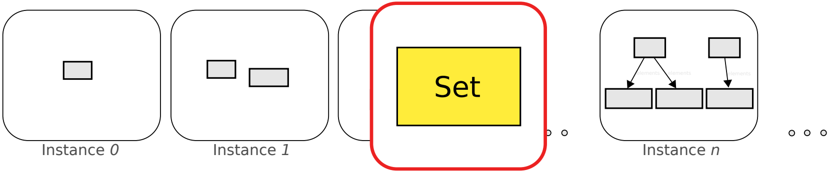

How to check refactorings?

- Remember: refactorings must not change the external behavior (behavior-preservation)
- *Refinement* guarantees substitutability
  - But might be irreversible
- Therefore, use refinement in "both directions"

Definition

Tow specifications A and C are *behavior-preserving*, iff $A \sqsubseteq C$ and $C \sqsubseteq A$.

Checking Refinement using downward simulation

1. *Init*:
   $\forall CState' \bullet CInit \Rightarrow \exists AState' \bullet AInit \wedge R'$
2. *Applicability*:
   $\forall AState; \; CState \bullet R \Rightarrow (\mathrm{pre}\, COp_i \Leftrightarrow \mathrm{pre}\, AOp_i)$
3. *Correctness*:
   $\forall AState; \; CState; \; CState' \bullet R \wedge COp_i \Rightarrow$
   $\exists AState' \bullet R' \wedge AOp_i$

Translate conditions into Alloy assertions

- Alloy allows direct translation, e.g.
- *Correctness*:

$$\forall AState;\ CState;\ CState' \bullet R \land COp_i \Rightarrow \exists AState' \bullet R' \land AOp_i$$

```
assert Correct {
all a: AState, c,c': CState| R[a,c] and COp_i =>
        {some a': AState| R[a',c'] and AOp_i}
}
```

Translate conditions into Alloy assertions

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  $$\forall AState;\ CState;\ CState' \bullet R \land COp_i \Rightarrow \exists AState' \bullet R' \land AOp_i$$

```
assert Correct {
all a: AState, c,c': CState| R[a,c] and COp_i =>
        {some a': AState| R[a',c'] and AOp_i}
}
```

- But, verification will **fail** due to the use of $\exists$ in the consequence of an implication

Problem with existential quantification

```
assert Closed {
all s0, s1: Set | some s2: Set |
s2.elements = s0.elements + s1.
     elements
}
```

```
some s0, s1: Set | all s2: Set |
not s2.elements = s0.elements +
     s1.elements
```

- Analyzer negates assertion
- Tries to find model for the negation

Problem with existential quantification

assert Closed {
all s0, s1: Set | **some** s2: Set |
s2.elements = s0.elements + s1.
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some s0, s1: Set | **all** s2: Set |
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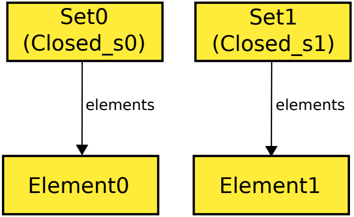
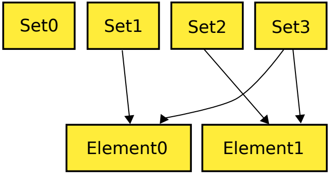
- Analyzer negates assertion
- Tries to find model for the negation
- Problem: actual instance of the model can be too small



Solutions to this problem?

- Constrain the model to fully populate the state space (*generator axiom*).

```
fact {
  some s: Set| no s.elements
  all s: Set, e: ELEMENT| some s':Set|
     s'.elements = s.elements + e }
```



- Analysis becomes intractable as scope explodes
  - To analyze *n* ELEMENT we need 2^n Set

- Instead: try to omit existential quantifier

Simplifying the refinement conditions

- A lot of refactorings do not change the state space
- Thus, representation relation R is the identity

- Given that R is total and bijective:
  $A \sqsubseteq_{DS} C$ and $C \sqsubseteq_{DS} A$ iff

1. *Init*:
   $\forall AState', CState' \bullet R' \Rightarrow (CInit \Leftrightarrow AInit)$
2. *Correctness*:
   $\forall AState;\ AState';\ CState;\ CState' \bullet R \wedge R' \Rightarrow (AOp_i \Leftrightarrow COp_i)$

Checking refactorings using the Alloy Analyzer

- Using the simplified conditions, we successfully checked refactorings
  - Inline Method
  - Substitute Algorithm
  - Extract Method
  - Rename
  - Consolidate Conditional Expression

Results

- Translation from Z into Alloy is mostly straight forward
  - Typical problems: integers, infinite data types, schema operators
- Use of existential quantifier is problematic
  - Found *workaround* to this problem when checking refactorings
- Open questions:
  - Does assumption of a total bijective representation relation prohibits the checking of practically relevant refactorings?
  - Compare performance of Alloy Analyzer with other verification tools.

Thank you for your attention!