On Relaxing Metric Information in Linear Temporal Logic

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***Abstract*—Metric LTL formulas rely on the *next* operator to encode time distances, whereas qualitative LTL formulas use only the *until* operator. This paper shows how to transform any metric LTL formula *M* into a qualitative formula *Q*, such that *Q* is satisfiable if and only if *M* is satisfiable over words with variability bounded with respect to the largest distances used in *M* (i.e., occurrences of *next*), but the size of *Q* is independent of such distances. Besides the theoretical interest, this result can help simplify the verification of systems with time-granularity heterogeneity, where large distances are required to express the coarse-grain dynamics in terms of fine-grain time units.**

***Keywords*-linear temporal logic; quantitative time; bounded variability; satisfiability**

I. Introduction and motivation

Linear temporal logic (LTL) supports a simple model of *metric time* through the *next* operator X. Under the assumption of a one-to-one correspondence between consecutive states and discrete instants of time, nested occurrences of X "count" instants to express time distances. LTL formulas without X — using only the *until* operator — are instead purely *qualitative*: they constrain the ordering of events, not their absolute distance. Therefore, qualitative LTL formulas express models that are insensitive to additions or removals of *stuttering steps*: consecutive repetitions of the same state. The fundamental properties of LTL with respect to its qualitative subset are well known from classic work: quantitative (metric) LTL is strictly more expressive [1], [2], [3], [4], but reasoning has the same worst-case complexity [5], [6].

The present paper investigates when the metric information, encoded by nested occurrences of X, is redundant and can be relaxed. The relaxation transforms a quantitative formula into an equi-satisfiable qualitative one that is independent of the number of X in the original formula; reasoning on the transformed formula is thus simpler by a factor proportional to the amount of metric information stripped.

The motivation behind this study refers to an informal notion of *redundancy*, which stuttering steps seem to encode. Consider a metric LTL formula ϕ describing models characterized by many stuttering steps distributed over large time distances; for example, the formalization of an event for elections that occur every four years in November, in a variable day of the month, with the day as time unit. Formula ϕ is large because it encodes large time distances in unary form with many occurrences of the X operator; for example, a four-year distance requires at least 1460 "next", one for each day. However, the information carried by ϕ is prominently redundant as every stuttering step is a duplication that only pads uneventful time instants. Is it possible, under a rigorous assumption of "sparse events", to simplify ϕ into an equi-satisfiable formula ϕ' which does not encode explicitly the redundant information?

The notion of *bounded variability*, adapted from dense-time models, provides a suitable formalization of the intuitive notion of "sparse events": models with bounded variability have, over every interval of fixed length, only a limited number v of steps that are not stuttering (i.e., redundant repetitions). The main result of the paper (in Section V) shows how to transform efficiently any LTL formula ϕ into a qualitative formula ϕ' such that ϕ is satisfiable *over models with bounded variability* iff ϕ' is satisfiable over models of any variability. The size of ϕ' does not depend on the distances (i.e., the number of nested occurrences of X) in ϕ but only on the maximum number of non-stuttering steps v. In other words, ϕ' drops some information encoded in ϕ; this information is not needed to decide satisfiability over models with bounded variability.

On the technical level, the construction that eliminates metric information relies on a normal form for LTL formulas and on discrete-time generalized versions of the dense-time *Pnueli operators* [7]. The correctness proof follows the idea of adding and removing stuttering steps to re-introduce the metric information dropped in models satisfying only qualitative constraints; it is reminiscent of the notion of *stretching*, also originally introduced for dense-time models [8], [9].

Besides the theoretical interest, the results of the present paper may be practically useful to simplify the temporal-logic analysis of systems characterized by heterogeneous components evolving over wildly different time scales, such as minutes, weeks, and years. Assuming incommensurable distances are not a concern, such heterogeneity of *time granularities* [10] can, in principle, be modeled in terms of the finest-grain time units; but this solution comes with a significant price to pay to accommodate the largest time units in terms of the smallest, resulting in huge formulas. If, however, the dynamics of the components with faster time scales are "sparse" enough, there is a redundancy in the global behavior of the system that the notion of bounded variability captures. Hence, the analysis can be carried out more efficiently by leveraging the results presented.

The paper is organized as follows. The rest of the present section recalls related work. Section II introduces notation and basic definitions. Section III presents normal forms for LTL formulas. Section IV discusses the equi-satisfiability of LTL and its qualitative subset. Section V shows how the metric information can be relaxed while preserving satisfiability, for models with bounded variability. Section VI outlines future work. For lack of space, the present version of the paper omits some details and several proofs: the presentation favors examples that elucidate the intuition behind the technicalities; a technical report [11] provides a complete presentation for reference.

Related work: The expressiveness and complexity of LTL and of its qualitative subset have been thoroughly investigated in the classic framework of temporal logic [12], [13], [14]. With respect to expressiveness, Lamport introduced the notion of *stuttering* to characterize qualitative LTL [1]; the characterization was completed by Peled and Wilke [2], perfected by Etessami and others [15], [3], [16], [17], and generalized by Kučera and Strejček [4]. With respect to complexity, the seminal work of Sistla and Clarke established the PSPACE-completeness of both LTL and qualitative LTL [5], and other authors have generalized or specialized the result [6], [18], [19].

To our knowledge, the present paper is the first investigating satisfiability-preserving relaxations of metric information in temporal logic formulas. More generally, the problem of formalizing systems with heterogeneous time granularities using temporal logic [10] has been studied by only a few authors [20], [21], [22], [23]; [22], in particular, presents an encoding of temporal granularities in LTL, but it does not discuss efficiency of the encoding.

Some of the techniques used in the the paper borrow from existing approaches in the literature. The normal forms for LTL introduced in Section III are related to a construction used in *temporal testers* [24]. The definition of *bounded variability* in Section II translates to discrete time a notion introduced for dense (or continuous) time models [25], [26], [27], [28].

Hirshfeld and Rabinovich studied the expressiveness and decidability of Pnueli operators over dense time [7]; the operators themselves were first mentioned in a conjecture attributed to Pnueli [29], [25]. Section V introduces discrete-time qualitative variants of such operators. *Counting operators* [30] are somehow similar to discrete-time Pnueli operators in that they both facilitate the expression of concise counting requirements; both extensions do not increase the expressive power of LTL, nor its complexity under a unary encoding. [31] introduce a much more expressive counting extension of LTL, which is decidable only in special cases.

II. Definitions

LTL syntax: LTL formulas are defined by:

$$\mathtt{LTL} \ni \phi ::= x \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \phi_1 \,\mathsf{U}\, \phi_2 \mid \mathsf{X}\,\phi$$

where x ranges over a set $\mathcal{P}$ of propositional letters. Assume the standard abbreviations for $\top, \bot, \vee, \Rightarrow, \Leftrightarrow$ and for the derived temporal operators: *eventually* $\mathsf{F}\phi \triangleq \top \,\mathsf{U}\, \phi$; *always*: $\mathsf{G}\phi \triangleq \neg\mathsf{F}\neg\phi$; *release*: $\phi_1 \,\mathsf{R}\, \phi_2 \triangleq \neg(\neg\phi_1 \,\mathsf{U}\, \neg\phi_2)$; *distance* $\mathsf{X}^k\phi = \underbrace{\mathsf{X}\,\mathsf{X}\cdots\mathsf{X}}_{k}\,\phi$ for $k \geq 0$.

Size and height: Let ϕ be an LTL formula. $\mathcal{P}(\phi) \subseteq \mathcal{P}$ denotes the (finite) set of propositional letters occurring in ϕ. $|\phi|$ denotes the size of ϕ. Three features determine the size of ϕ: the size $|\phi|_{\mathsf{p}}$ of its propositional structure; the size $|\phi|_{\mathsf{U}}$ of its *until* subformulas; and the size $|\phi|_{\mathsf{X}}$ of its *next* subformulas; they are defined inductively as follows.

$$|\phi|_{\mathsf{p}} = \begin{cases} 1 & \phi = x \\ 1 + |\phi'|_{\mathsf{p}} & \phi = \neg\phi' \\ 1 + |\phi_1|_{\mathsf{p}} + |\phi_2|_{\mathsf{p}} & \phi = \phi_1 \wedge \phi_2 \\ |\phi_1|_{\mathsf{p}} + |\phi_2|_{\mathsf{p}} & \phi = \phi_1 \,\mathsf{U}\, \phi_2 \\ |\phi'|_{\mathsf{p}} & \phi = \mathsf{X}\,\phi' \end{cases}$$

$$|\phi|_{\mathsf{U}} = \begin{cases} 0 & \phi = x \\ |\phi'|_{\mathsf{U}} & \phi = \neg\phi' \\ |\phi_1|_{\mathsf{U}} + |\phi_2|_{\mathsf{U}} & \phi = \phi_1 \wedge \phi_2 \\ 1 + |\phi_1|_{\mathsf{U}} + |\phi_2|_{\mathsf{U}} & \phi = \phi_1 \,\mathsf{U}\, \phi_2 \\ |\phi'|_{\mathsf{U}} & \phi = \mathsf{X}\,\phi' \end{cases}$$

$$|\phi|_{\mathsf{X}} = \begin{cases} 0 & \phi = x \\ |\phi'|_{\mathsf{X}} & \phi = \neg\phi' \\ |\phi_1|_{\mathsf{X}} + |\phi_2|_{\mathsf{X}} & \phi = \phi_1 \wedge \phi_2 \\ |\phi_1|_{\mathsf{X}} + |\phi_2|_{\mathsf{X}} & \phi = \phi_1 \,\mathsf{U}\, \phi_2 \\ 1 + |\phi'|_{\mathsf{X}} & \phi = \mathsf{X}\,\phi' \end{cases}$$

Correspondingly, $|\phi|$ is $|\phi|_{\mathsf{p}} + |\phi|_{\mathsf{U}} + |\phi|_{\mathsf{X}}$.

For a temporal operator $\mathsf{H} \in \{\mathsf{U}, \mathsf{X}\}$, the *temporal height* (or *nesting depth*) $\mathcal{H}(\phi, \mathsf{H})$ of H in ϕ is the maximum number of nested occurrences of H in ϕ. For example, $\mathcal{H}(\phi, \mathsf{X}) = 0$ iff X is not used in ϕ. $d(\phi)$ denotes instead the maximum number of *consecutive* nested occurrences of the *next* operator, that is the largest n such that X^n occurs in ϕ; clearly, $d(\phi) \leq \mathcal{H}(\phi, \mathsf{X})$. Finally, $s(\phi)$ is the number of *distinct subformulas* of the form $\mathsf{X}^m\phi$ with $m \geq 1$. Notice that $|\phi|_{\mathsf{X}}$ is bounded by $d(\phi) \cdot s(\phi)$, hence $|\phi|$ is in $\mathrm{O}\left(|\phi|_{\mathsf{p}} + |\phi|_{\mathsf{U}} + d(\phi) \cdot s(\phi)\right)$.

$\mathtt{L}\big(\mathsf{U}^{h_1}, \mathsf{X}^{h_2}\big)$ denotes the fragment of LTL whose formulas ψ are such that $\mathcal{H}(\psi, \mathsf{U}) \leq h_1$ and $\mathcal{H}(\psi, \mathsf{X}) \leq h_2$. Omit the superscript to mean that there is no bound on the temporal height of an operator. Hence, $\mathtt{L}(\mathsf{U}, \mathsf{X})$ is the same as all LTL; $\mathtt{L}\big(\mathsf{U}, \mathsf{X}^0\big) = \mathtt{L}(\mathsf{U})$ denotes *qualitative* LTL, where no *next* operator is used; and $\mathtt{L}\big(\mathsf{U}^0, \mathsf{X}^0\big) = \mathtt{P}(\mathcal{P})$ denotes *propositional formulas* without any temporal operator.

ω-words: An *ω-word* (or simply *word*) over a set S of propositional letters is a mapping $w : \mathbb{N} \to 2^S$ or, equivalently, a denumerable sequence $w(0)w(1)\cdots$ of elements $w(i) \subseteq S$. The set of all ω-words over S is denoted by $\mathcal{W}[S]$. For $T \subseteq S$, $w|_T$ is the projection of w over T, defined as $w(0)|_T w(1)|_T \cdots$, where $w(i)|_T = w(i) \cap T$ for all $i \in \mathbb{N}$. The projection is extended to sets of words as expected. For $i, j \in \mathbb{N}$, w_i denotes the suffix $w(i)w(i+1)\cdots$ of w; $w(i,j)$ denotes the subword of w of length j starting at $w(i)$ (with $w(i,0) = \epsilon$ for all i); and $w(i{:}j)$ denotes the subword $w(i)w(i+1)\cdots w(j)$ (with $w(i,j) = \epsilon$ for all $j < i$).

LTL semantics: The satisfaction relation $\models$ is defined as usual, for an LTL formula ϕ, interpreted over an ω-word w over $\mathcal{P}$, at position $i \in \mathbb{N}$.

$w, i \models p$	iff	$p \in w(i)$
$w, i \models \neg\phi$	iff	$w, i \not\models \phi$
$w, i \models \phi_1 \wedge \phi_2$	iff	$w, i \models \phi_1$ and $w, i \models \phi_2$
$w, i \models \phi_1 \mathsf{U} \phi_2$	iff	for some $j \geq i$, $w, j \models \phi_2$ and for all $i \leq k < j$, $w, k \models \phi_1$
$w, i \models \mathsf{X}\phi$	iff	$w, i+1 \models \phi$
$w \models \phi$	iff	$w, 0 \models \phi$

$[\![\phi]\!]$ denotes the set $\{w \in \mathcal{W}[\mathcal{P}] \mid w \models \phi\}$ of all models of ϕ. ϕ is *satisfiable* iff $[\![\phi]\!] \neq \emptyset$ and is *valid* iff $[\![\phi]\!] = \mathcal{W}[\mathcal{P}]$. Two formulas ϕ_1, ϕ_2 are *equivalent* iff $[\![\phi_1]\!] = [\![\phi_2]\!]$; they are *equi-satisfiable* iff they are either both satisfiable or both unsatisfiable. Satisfiability is PSPACE-complete for LTL formulas and qualitative LTL formulas.

Stuttering: A position $i \in \mathbb{N}$ is *redundant* in a word w iff $w(i+1) = w(i)$ and there exists a $j > i$ such that $w(j) \neq w(i)$; a redundant position is also called *stuttering step*. Conversely, a *non-stuttering step (nss)* is any position i such that $w(i+1) \neq w(i)$ or $w(i+j) = w(i)$ for all $j \in \mathbb{N}$. A *stutter-free* word is one without stuttering steps. Two words w_1, w_2 are *stutter-equivalent* (or equivalent under stuttering) iff they are reducible to the same stutter-free word by removing an arbitrary number of stuttering steps. A set of words W is *closed under stuttering* (or stutter-invariant) iff for every word $w \in W$, for all words w' such that w and w' are stutter-equivalent, $w' \in W$ too.

Proposition 1 (Stuttering and qualitative LTL [1], [2])**.** *Closure under stutter equivalence is a necessary and sufficient condition for qualitative* LTL *languages.*

Variability: Let W be a set of words and v, k two positive integers. A set of propositional letters $P \subseteq \mathcal{P}$ has *variability bounded by v/k in W* iff: for every $w \in W$, the projection $w(i,k)|_P$ over P of every subword $w(i,k)$ of length k has at most v nss. $\mathbf{var}(P, v/k)$ denotes the set of all words where P has variability bounded by v/k. Note that $\mathbf{var}(P, v/k)$ is not closed under stuttering for any $v < k$.

Example 2 (The elections)**.** Consider elections that occur every four years, in one of two consecutive days. The example is deliberately kept simple to be able to demonstrate it with the various constructions of the paper. Proposition q marks the first day of every quadrennial, hence it holds initially and then precisely every $d_4 = 365 \cdot 4 = 1460$ days. The elections e occur once within every quadrennial; precisely they occur $d_2 = 40$ or $d_3 = 41$ days before the end of the quadrennial. Assuming models with variability bounded by $5/1460$, the behavior is completely described by the following formula.

$$\begin{aligned}
& q && (1)\\
& \wedge\ \mathsf{G}\left(q \Rightarrow \mathsf{X}\,(\neg q \wedge \neg q \,\mathsf{U}\, q) \wedge \mathsf{X}^{d_4} q\right) && (2)\\
& \wedge\ \mathsf{G}\left(q \Rightarrow \mathsf{X}\,\neg\,(\neg e \,\mathsf{U}\, q)\right) && (3)\\
& \wedge\ \mathsf{G}\left(e \Rightarrow \neg q \ \wedge\ \mathsf{X}\,(\neg e \,\mathsf{U}\, q)\right) && (4)\\
& \wedge\ \mathsf{G}\left(e \Rightarrow \mathsf{X}^{d_2} q \vee \mathsf{X}^{d_3} q\right) && (5)
\end{aligned}$$

The proposition q marks the beginning of every quadrennial: q holds initially (1) and then always *at least* every d_4 steps (2). The elections, marked by proposition e, must occur once before the next quadrennial starts (3). They must also occur not at the beginning of a new quadrennial and at most once during the quadrennial (4); precisely, they occur d_2 or d_3 days before the end of the current quadrennial (5). A variability of $5/1460$ makes such model tight, as it allows *at most* 5 nss over a windows of length 1460: 2 of them accounts for q becoming true and then false again once, and the other 3 nss mark a similar double transition of e. ◊

III. Separated-next form for LTL

A formula in *separated-next form* (SNF) is written as:

$$\kappa \ \wedge \mathsf{G}\left(\bigwedge_{i=1,\ldots,M} (x_i \Leftrightarrow \mathsf{X}^{\mathbb{D}(i)} \pi_i)\right) \qquad (6)$$

where $\kappa \in \mathtt{L}(\mathsf{U})$, $x_i \in \mathcal{P}$, $\pi_i \in \mathtt{P}(\mathcal{P})$, and $\mathbb{D}$ is a monotonically non-decreasing mapping $[1..M] \to \mathbb{N}_{>0}$.

Lemma 3. *For any $\phi \in$ LTL it is possible to build, in polynomial time, an equi-satisfiable formula η in SNF (6) such that $|\kappa|$, $\max_i |\pi_i|$, and $|\mathcal{P}(\eta)|$ are in $\mathrm{O}(|\phi|_{\mathsf{p}} + |\phi|_{\mathsf{U}} + s(\phi))$, $M = s(\phi)$, and $d(\eta) = \max_i \mathbb{D}(i) = \mathbb{D}(M) = d(\phi)$.*

Example 4. The following formula Ω is the formula of Example 2 in separated-next form, with $d_1 = d_2 = 1$, $d_3 = 40$, $d_4 = 41$, $d_5 = 1460$.

$$\Omega \triangleq \underbrace{\left(q \wedge \mathsf{G}\begin{pmatrix} (u \Leftrightarrow \neg e \,\mathsf{U}\, q) \\ \wedge\,(v \Leftrightarrow \neg q \wedge \neg q \,\mathsf{U}\, q) \\ \wedge\,(q \Rightarrow x_2 \wedge x_5) \wedge (q \Rightarrow \neg x_1) \\ \wedge\,(e \Rightarrow \neg q \ \wedge\ x_1) \wedge (e \Rightarrow x_3 \vee x_4) \end{pmatrix}\right)}_{\kappa_\Omega} \wedge\, \mathsf{G}\begin{pmatrix} (x_1 \Leftrightarrow \mathsf{X}^{d_1} u) \\ \wedge\,(x_2 \Leftrightarrow \mathsf{X}^{d_2} v) \\ \wedge\,(x_3 \Leftrightarrow \mathsf{X}^{d_3} q) \\ \wedge\,(x_4 \Leftrightarrow \mathsf{X}^{d_4} q) \\ \wedge\,(x_5 \Leftrightarrow \mathsf{X}^{d_5} q) \end{pmatrix} \qquad (7)$$

Notice that $\kappa_\Omega \in \mathtt{L}(\mathsf{U})$ is the first conjunct, $|\mathcal{P}(\Omega)| = 9$, $M_\Omega = 5$, $d(\Omega) = d_5$; the last one dominates over the other size parameters. In any model of Ω corresponding to a model of (1–5) with variability $5/1460$ there are at most 6 nss over $[1..1460]$: $1, 1419, 1420, 1421, 1459, 1460$, corresponding to a variability of $6/1460$. $\Diamond$

IV. Reducing LTL to qualitative LTL

This section outlines a transformation of LTL formulas into equi-satisfiable $\mathtt{L}(\mathsf{U})$ formulas of polynomially correlated size. The feasibility of the construction is unsurprising in hindsight, given the complexity results about qualitative LTL [5] and Etessami's construction [3]. However, it is the necessary basis of the techniques used to derive the main result for models with bounded variability, hence we succinctly present it here.

Theorem 5. *Given an* LTL *formula η, it is possible to build, in polynomial time, a qualitative* LTL *formula $\xi \in \mathtt{L}(\mathsf{U})$ such that η and ξ are equi-satisfiable and have polynomially correlated size.*

Informally, the construction from an LTL formula η into an equi-satisfiable qualitative ξ works as follows. Introduce a fresh propositional letter s. Constrain s to change truth value with any propositional letter in $\mathcal{P}$; in other words, any nss coincides with a nss of s. Then, replace any occurrence of a subformula $\mathsf{X}\, p$ with a suitable *until* formula that defines the value of p at the next nss of s. In practice, this means that a formula such as $\mathsf{X}\, p$ forces p to hold in the next state (with a new state of s) only if this is necessary, i.e., if this requires a nss. This changes the quantitative $\mathsf{X}\, p$ formula into a qualitative formula where the precise metric information is relaxed.

The formal construction uses the following abbreviations, also employed in Section V, for every propositional formula $\pi \in \mathtt{P}(\mathcal{P})$, and every $\phi \in \mathtt{LTL}$:

$$\curlywedge(\pi) \triangleq \pi \wedge \mathsf{F}\neg\pi \;\Rightarrow \begin{pmatrix} s\,\mathsf{U}\,(\neg\pi \wedge \neg s) \\ \vee \\ \neg s\,\mathsf{U}\,(\neg\pi \wedge s) \\ \vee \\ \bigvee_{q\in\mathcal{P}\setminus\{\pi\}} (q \wedge \pi)\,\mathsf{U}\,(\neg q \wedge \pi) \\ \vee \\ \bigvee_{q\in\mathcal{P}\setminus\{\pi\}} (\neg q \wedge \pi)\,\mathsf{U}\,(q \wedge \pi) \end{pmatrix}$$

$$\curlyvee(\mathcal{P}) \triangleq \left(\bigwedge_{p\in\mathcal{P}} \mathsf{G}p \vee \mathsf{G}\neg p\right) \Rightarrow \mathsf{G}s$$

$$\mathcal{U}(\phi) \triangleq s\,\mathsf{U}\,\phi \;\vee\; \neg s\,\mathsf{U}\,\phi$$

$$\mathcal{R}(\phi) \triangleq (\phi \wedge s \;\Rightarrow\; \neg s\,\mathsf{R}\,\phi) \;\wedge\; (\phi \wedge \neg s \;\Rightarrow\; s\,\mathsf{R}\,\phi)$$

$$\mathcal{X}(\phi) \triangleq \mathcal{U}(\phi) \wedge \mathcal{R}(\phi)$$

$\curlywedge(\pi)$ links any transition of the truth value of π to occur simultaneously with a transition of s. $\curlyvee(\mathcal{P})$ deals with the special case where no proposition ever changes truth value. $\mathcal{X}(\pi)$ is instead essentially a qualitative relaxations of the *next* operator: $w, i \models \mathcal{X}(p)$ holds iff the next nss of s is $j \geq i$ and $w, j+1 \models p$ holds.

V. LTL with bounded variability

This section shows how to more succinctly encode the redundancy of stuttering steps in words with bounded variability. The results require $\mathtt{LTL}(\mathsf{U}, \mathsf{exqPn})$: an extension of $\mathtt{L}(\mathsf{U})$ with a qualitative variant of the Pnueli operators.

A. Pnueli operators

The results of the present paper are based on a qualitative discrete-time version of the Pnueli operators: the *qualitative extended Pnueli operators* $\mathsf{exqPn}_k^{n;\langle n_1,\ldots,n_k\rangle}$ for $k, n \in \mathbb{N}$ and $n_1, \ldots, n_k \in \mathbb{N} \cup \{*\}$. Their semantics is defined as follows: $w, i \models \mathsf{exqPn}_k^{n;\langle n_1,\ldots,n_k\rangle}(\phi_1, \ldots, \phi_k)$ holds iff there exist k positions $i \leq k_1 < \cdots < k_k$ such that all the following hold, for all $1 \leq j \leq k$: (1) k_j is a nss; (2) $w, k_j + 1 \models \phi_j$; (3) for $j > 1$, if $n_j \neq *$ then there are no more than n_j nss between k_{j-1} and $k_j - 1$ (both included); (4) if $n_1 \neq *$ then there are no more than n_1 nss between i and k_1 (both included); (5) there are no more than n nss between i and k_k (both included). For example, if $n_1 = 1$, ϕ_1 must hold right after the first nss that follows or is at i, independently of the other following $k-1$ nss.

Example 6. Consider the word w:

$\underline{\mathbf{1}}$	2	3	4	5	$\underline{\mathbf{6}}$	$\underline{\mathbf{7}}$	8	9	$\underline{\mathbf{10}}$	11	$\underline{\mathbf{12}}$	$\underline{\mathbf{13}}$	14
s	$\overline{s}$	$\overline{s}$	$\overline{s}$	$\overline{s}$	$\overline{s}$	s	$\overline{s}$	$\overline{s}$	$\overline{s}$	s	s	$\overline{s}$	s
$\overline{v}$	v	v	v	v	v	$\overline{v}$	v	v	v	v	v	$\overline{v}$	v
q	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	q
$\overline{e}$	$\overline{e}$	$\overline{e}$	$\overline{e}$	$\overline{e}$	$\overline{e}$	$\overline{e}$	e	e	e	$\overline{e}$	$\overline{e}$	$\overline{e}$	$\overline{e}$

where nss are in bold and underlined, and $\overline{x}$ denotes $\neg x$. For the positions $1, 6, 7, 13$, $w, 1 \models \mathsf{exqPn}_4^{6;\langle 3,2,*,3\rangle}(v, \neg q, e, q)$ holds. On the contrary, $w, 1 \not\models \mathsf{exqPn}_4^{6;\langle 3,2,*,1\rangle}(v, \neg q, e, q)$; in fact, let $k_1, \ldots, k_4$ be the positions that match the semantics of the operator. Then, $k_4 = 13$ as q only holds at 14, so that the last component of the constraint $\langle 3, 2, *, 1\rangle$ forces k_3 to be 12, the nss immediately before 13; but $w, 12+1 \not\models e$. $\Diamond$

$\mathtt{LTL}(\mathsf{U}, \mathsf{exqPn})$ is the extension of qualitative LTL with qualitative extended Pnueli operators. Any $\mathtt{LTL}(\mathsf{U}, \mathsf{exqPn})$ formula has an equi-satisfiable $\mathtt{L}(\mathsf{U})$ formula of polynomially correlated size that can be built in polynomial time, when the integer constants used in the Pnueli operators use a unary encoding. Precisely, to encode a formula $\mathsf{exqPn}_k^{n;\langle n_1,\ldots,n_k\rangle}(\phi_1, \ldots, \phi_k)$ introduce n^2 letters $\{q_i^j \mid 1 \leq i, j \leq n\}$. Every q_i^j holds iff $\mathsf{exqPn}_i^{j;\langle n_1,\ldots,n_i\rangle}(\phi_1, \ldots, \phi_i)$ does; then, a formula of size $\mathrm{O}(n \cdot \max_i |\phi_i|)$ defines each q_i^j.

The construction outlined is general, but the remainder will use $\mathtt{LTL}(\mathsf{U}, \mathsf{exqPn})$ formulas Λ over $\mathcal{Q} = \mathcal{P} \cup \{s\}$ in the form:

$$\Lambda \triangleq \curlyvee(\mathcal{P}) \wedge \bigwedge_{p\in\mathcal{P}} \mathsf{G}\begin{pmatrix}\curlywedge(p)\wedge\\ \curlywedge(\neg p)\end{pmatrix} \wedge \kappa \wedge \bigwedge_{i=1,\dots,M} \mathsf{G}\left(\xi_i \Rightarrow \mathsf{exqPn}_{\mathbb{I}(i)}^{\mathbb{J}(i);\langle\mathbb{K}(i)\rangle}\left(\psi_1^i,\dots,\psi_{\mathbb{I}(i)}^i\right)\right) \quad (8)$$

where $\mathbb{J}, \mathbb{I}$ are two mappings $[1..M] \to \mathbb{N}_{>0}$; $\mathbb{K}$ is a mapping $[1..M] \to (\mathbb{N} \cup \{*\})^{\mathbb{I}(i)}$; $\kappa, \psi_i \in \mathsf{L}(\mathsf{U})$ for all $1 \leq i \leq M$; $\xi_i \in \mathsf{P}(\mathcal{P})$; and s does not occur in κ or ψ_i^j.

Lemma 7. *$[\![\Lambda]\!]$ is closed under stuttering.*

Lemma 8. *It is possible to build, in polynomial time, a formula Λ' such that: (1) $\Lambda' \in \mathsf{L}(\mathsf{U})$; (2) $|\Lambda'|$ is polynomially bounded by $|\Lambda|$; (3) Λ' and Λ are equi-satisfiable.*

B. Relaxing distance formulas

Consider a generic formula ϕ and let η be ϕ in SNF; the following construction builds a $\phi' \in \mathtt{LTL}(\mathsf{U}, \mathsf{exqPn})$ from η such that Lemma 11 holds. Theorem 9 follows after transforming ϕ' into ϕ'' by eliminating the qualitative extended Pnueli operators according to Lemma 8.

Theorem 9. *Given an* LTL *formula ϕ and an integer parameter $\mathbb{V} > 0$, it is possible to build, in polynomial time, a qualitative* LTL *formula ϕ'' such that: (1) $|\phi''|$ is polynomial in $\mathbb{V}, |\phi|_{\mathsf{p}}, |\phi|_{\mathsf{U}}, s(\phi)$ but is independent of $d(\phi)$; (2) ϕ is satisfiable over words in $\mathbf{var}(\mathcal{Q}, \mathbb{V}/d(\phi))$ iff ϕ'' is satisfiable over unconstrained words.*

Informal presentation: The basic idea consists of relaxing every distance formula $\mathsf{X}^d a$ into a qualitative formula $\mathcal{X}^{d'}(a)$ with $d' \leq \mathbb{V}$, so that consecutive nss take the role of consecutive positions. The elimination or addition of stuttering steps reconciles words in the quantitative and qualitative transformed formulas. For example, $w, 1 \models \mathcal{X}^6(q)$ holds because q holds at position 14; adding $41 - 14 = 27$ repetitions of position 2 transforms w into a word w' where the quantitative requirement $w', 1 \models \mathsf{X}^{41} q$ holds as well.

The transformation must also preserve the ordering among events: if $\mathsf{X}^d a$ and $\mathsf{X}^e b$ both hold for some $d < e$, then $\mathcal{X}^{d'}(a)$ and $\mathcal{X}^{e'}(b)$ should hold for suitable $d' < e'$. Another constraint requires that $e' - d' \leq e - d$; otherwise, the transformed formula admits words with $e' - d' > e - d$ non-stuttering steps between consecutive occurrences of a and b, which may not be removable to put a and b at an absolute distance of $e - d$. For example, $\mathcal{X}(s) \wedge \mathcal{X}^2(q)$ is a suitable relaxation of $\mathsf{X}^{30} s \wedge \mathsf{X}^{31} q$, whereas $\mathcal{X}(s) \wedge \mathcal{X}^3(q)$ is not: $w, 10 \models \mathcal{X}(s) \wedge \mathcal{X}^3(q)$ but the nss 13 makes it impossible to pad w with stuttering steps such that s and q hold at positions $10 + 30$ and $10 + 31$.

Using these ideas, a formula η in SNF (6) is transformed by replacing the distance formulas with qualitative "snapshots" using the qualitative extended Pnueli operators: the predicates $\pi_1, \pi_2, \dots, \pi_M$ hold orderly over some of the the following $\mathbb{V}$ nss, with the additional constraint that, between any two consecutive π_i, π_{i+1}, no more nss than the difference of the corresponding distances occur. For example, if $\neg x_1 \wedge x_2 \wedge \neg x_3 \wedge \neg x_4 \wedge x_5$ (corresponding to predicates $\neg u, v, \neg q, \neg q, q$) then formula $\mathsf{exqPn}_4^{6;\langle 1,*,1,*\rangle}(\neg u \wedge v, \neg q, \neg q, q)$ must hold, where $\neg u \wedge v$ occurs after the next nss and $\neg q, \neg q$ occupy consecutive nss.

This approach can be made rigorous, but introduces an exponential blow-up because it considers each of the 2^M subset of propositions $x_1, \dots, x_M$. The following construction avoids this blow-up by introducing auxiliary propositions y_i's and z_i^j's that mark nss and decouple them from the propositions that must hold therein.

Each y_i holds precisely from the i-th nss until the next $1 + (i \bmod \mathbb{V})$ nss. Then, for each given h, the propositions $z_1^h, z_2^h, \dots, z_m^h$ (where m is the number of different distances used in η) hold sequentially and cyclically from when y^h holds. Each z_k^h marks a position in the sequence that satisfies the qualitative extended Pnueli operator under consideration; correspondingly, for each index k' corresponding to a distance with index k, $\pi_{k'}$ holds with z_k^h iff $x_{k'}$ holds with y_h. For example, if $\neg x_1 \wedge x_2 \wedge \neg x_3 \wedge \neg x_4 \wedge x_5$ holds when some y_k holds, then the corresponding predicates $\neg u \wedge v, \neg q, \neg q, q$ hold orderly with the next occurrences of $z_1^k, z_2^k, z_3^k, z_4^k$.

Detailed construction: Consider a generic LTL formula η in the SNF of Formula (6). Introduce $\mathbb{V}$ letters $\{y_i \mid 1 \leq i \leq \mathbb{V}\}$. Formula $\varkappa\langle y\rangle$ constrains y_i to occur synchronously with every i-th nss:

$$\varkappa\langle y\rangle \triangleq y_1 \wedge \bigwedge_{1\leq i\leq\mathbb{V}} \mathsf{G}\left(y_i \Rightarrow \begin{pmatrix}\mathcal{X}(y_{1+(i \bmod \mathbb{V})})\\ \bigwedge_{j\neq i} \neg y_j \wedge\\ y_i \,\mathsf{U}\, y_{1+(i \bmod \mathbb{V})}\end{pmatrix}\right) \quad (9)$$

Let $D_1, D_2, \dots, D_m$ be the sequence of sets that partition $[1..M]$ in such a way that indices involving the same number of consecutive nested X's are in the same set, and the sets appear in the sequence in increasing order of nested X's; formally: $i, j \in D_k$ with $k \triangleq D(i) = D(j)$ for some k iff $\mathbb{D}(i) = \mathbb{D}(j) \triangleq d_k$ (and d_0 is defined as 0); and $i \in D_{k_1}$ and $j \in D_{k_2}$ with $k_1 < k_2$ implies $\mathbb{D}(i) < \mathbb{D}(j)$.

Then, introduce another $m \cdot \mathbb{V}$ letters $\{z_i^j \mid 1 \leq i \leq m, 1 \leq j \leq \mathbb{V}\}$. At every i-th nss, marked by y_i, the sequence $z_1^j, \dots, z_m^j$ must hold over m of the following $\mathbb{V}$ nss; moreover, between each z_j^i and its preceding z_{j-1}^i there must be no more than $d_i - d_{i-1}$ nss, unless $d_i - d_{i-1} > \mathbb{V} - i + 1$. After defining, for $1 \leq i \leq m$:

$$\delta_i \quad \triangleq \quad \begin{cases} d_i - d_{i-1} & \text{if } d_i - d_{i-1} \leq \mathbb{V} - i + 1 \\ * & \text{otherwise} \end{cases}$$

the qualitative extended Pnueli operators capture this behavior of the z_i^j's.

$$\varkappa\langle z, \mathsf{Pn}\rangle \triangleq \bigwedge_{1\leq i\leq\mathbb{V}} \mathsf{G}\left(y_i \Rightarrow \mathsf{exqPn}_m^{\mathbb{V};\langle\delta_1,\dots,\delta_m\rangle}(z_1^i, \dots, z_m^i)\right) \quad (10)$$

Additionally, constrain the z_i^j's to hold sequentially, according to the following.

$$\varkappa\langle z, \mathsf{U}\rangle \triangleq \left(\bigwedge_{h,j\neq 1} \neg z_h^j\right) \mathsf{U}\, z_1^1 \,\wedge$$
$$\bigwedge_{\substack{1\leq j\leq \mathbb{V}\\ 1\leq i\leq m}} \mathsf{G}\left(z_i^j \Rightarrow \begin{pmatrix} \neg z_{1+(i \bmod m)}^{1+(j \bmod \mathbb{V})} \\ \wedge \bigwedge_{h\neq i} \neg z_h^j \end{pmatrix} \mathsf{U}\, z_{1+(i \bmod m)}^j\right) \quad (11)$$

Once the z_i^j's and the y_i's are constrained, link the x_i's to the values of the π_i's in the distance formulas. If some x_i holds, after or at the j-th nss and before the $j+1$-th, then π_i has to hold at the k-th position in the sequence $z_1^j, \ldots, z_m^j$, with $k = D(i)$.

$$\varkappa\langle x, \pi\rangle \quad \triangleq$$
$$\bigwedge_{\substack{1\leq i\leq M\\ 1\leq j\leq \mathbb{V}}} \mathsf{G}\begin{pmatrix} x_i \wedge y_j \Rightarrow \neg z_{D(i)}^j \,\mathsf{U}\, z_{D(i)}^j \wedge \pi_i \\ \wedge \\ \neg x_i \wedge y_j \Rightarrow \neg z_{D(i)}^j \,\mathsf{U}\, z_{D(i)}^j \wedge \neg\pi_i \end{pmatrix} \quad (12)$$

Finally, combine the various $\varkappa$ formulas to transform η into ϕ':

$$\phi' \triangleq \kappa \wedge \curlyvee(\mathcal{Q}) \wedge \bigwedge_{p\in\mathcal{Q}} \mathsf{G}\begin{pmatrix} \curlywedge(p)\wedge \\ \curlywedge(\neg p) \end{pmatrix} \wedge \varkappa\langle y\rangle \,\wedge$$
$$\wedge\, \varkappa\langle z, \mathsf{Pn}\rangle \wedge \varkappa\langle z, \mathsf{U}\rangle \wedge \varkappa\langle x, \pi\rangle \quad (13)$$

Example 10. In the elections example, $\mathbb{V} = 6$, $m = 4$ instantiate $\varkappa\langle y\rangle$, $\varkappa\langle z, \mathsf{U}\rangle$, and $\varkappa\langle x, \pi\rangle$. Then, $\delta_1 = \delta_3 = 1$ and $\delta_2 = \delta_4 = *$ instantiate $\varkappa\langle z, \mathsf{Pn}\rangle$. ♢

The correctness of the above construction and the proof of Theorem 9 rely on the following two lemmas.

Lemma 11. *η is satisfiable over words in $\mathbf{var}(\mathcal{Q}, \mathbb{V}/d(\phi))$ iff ϕ' is satisfiable over unconstrained words.*

Proof: Let D be $d(\eta)$, which equals $d(\phi)$ by Lemma 3.

$\mathbf{SAT}(\eta) \Rightarrow \mathbf{SAT}(\phi')$. Let $w \in \mathbf{var}(\mathcal{Q}, \mathbb{V}/D)$ such that $w \models \eta$. w' adds propositions s, y_i, z_i^j, constrained as follows. s switches its truth value at every nss, except for possibly an infinite tail of constant values over w. Exactly one of the y_i's holds at every instant, and they rotate at every nss signaled by s. Whenever a given y_j holds, a sequence of z_i^j's hold over the following $\mathbb{V}$ nss, in a sequential fashion. Namely, let k be the first step where a certain y_j holds, let h_i be the last non-stuttering before position $k + d_i$, and let l_i be the δ_i-th nss after h_{i-1} (included, with $h_0 = k$); then, z_i^j starts to hold at $\min(h_i, l_i, \mathbb{V}-k+1)+1$, and holds until the next z_{i+i}^j.

Once w' is built, the rest of the proof relies on some of the notions introduced in Section IV. It is clear that $w' \models \bigwedge_{p\in\mathcal{Q}} \curlyvee(\mathcal{P}) \wedge \mathsf{G}(\curlywedge(p) \wedge \curlywedge(\neg p))$ and $w' \models \kappa$. In addition, $w' \models \varkappa\langle y\rangle \wedge \varkappa\langle z, \mathsf{U}\rangle$ is a consequence of the set up of the y_j's and the z_i^j's. Then, let i be the current generic instant and $b \subseteq [1..M]$ be a generic subset such that $\bigwedge_{i\in b} x_i \wedge \bigwedge_{i\notin b} \neg x_i$ holds at i. Hence, $w, i \models \mathsf{X}^{\mathbb{D}(j)}\pi_j$ holds for all $j \in b$ and $w, i \models \mathsf{X}^{\mathbb{D}(k)}\neg\pi_k$ holds for all $k \notin b$. The variability of w — and that of w' — is bounded by $\mathbb{V}/D$; hence, there are at most $\mathbb{V}$ nss of item s over positions i to $i + D$. Let $i \leq t_1 < \cdots < t_{\mathbb{V}} \leq i+D$ be these transition instants. There are only stuttering steps between any such two consecutive t_i's, hence there exists a subset $u_1 < \cdots < u_m$ of the t_i's such that z_i^j holds at u_i for all i's and some unique j. Now, for all g such that $D(g) = i$, π_g holds at $k + d_i$ and (at least) since the previous and until the next nss. Because of how each z_i^j's mark the stuttering positions before $k + d_i$, for every g such that $D(g) = i$, π_i must in particular hold where z_i^j first holds; because i is generic, $w' \models \varkappa\langle x, \pi\rangle$ holds. Also, if $d_i - d_{i-1} \leq \mathbb{V} - i + 1$, there are no more than $d_i - d_{i-1}$ nss between u_{i-1} and u_i, for all $1 \leq i \leq m$ (and assuming $u_0 = d_0 = 0$); this establishes $w', i \models \mathsf{exqPn}_m^{\mathbb{V};\langle\delta_1,\ldots,\delta_m\rangle}\left(z_1^j, \ldots, z_m^j\right)$. In all, $w' \models \phi'$ holds.

$\mathbf{SAT}(\phi') \Rightarrow \mathbf{SAT}(\eta)$. Let w' be an unconstrained word such that $w' \models \phi'$. Initially, let w be w' with all stuttering steps removed; $w \models \phi'$ as well from Lemma 7. Modify w as follows, until $w \models \eta$ is the case.

Let i be the current generic instant and $b \subseteq [1..M]$ be a generic subset such that $\bigwedge_{i\in b} x_i \wedge \bigwedge_{i\notin b} \neg x_i$ holds at i on w. The rest of the proof works inductively on $1 \leq h \leq M$; let us focus on the more interesting inductive step.

Let $i \leq t_1 < \cdots < t_{\mathbb{V}}$ be the following $\mathbb{V}$ nss of s — and hence of any proposition in $\mathcal{Q}$ as well, according to $\curlyvee(\mathcal{Q}) \wedge \bigwedge_{p\in\mathcal{Q}} \mathsf{G}(\curlywedge(p) \wedge \curlywedge(\neg p))$. $\varkappa\langle y\rangle$ implies that a unique y_j holds at i; correspondingly, $\varkappa\langle z, \mathsf{Pn}\rangle$ entails that there exists a subset of the $u_1 < \cdots < u_m$ of the sequence $t_1 < \cdots < t_{\mathbb{V}}$ such that z_k^j holds at $u_k + 1$ for all $1 \leq k \leq m$. Assume x_h holds at i (the case of $\neg x_h$ is clearly symmetrical and is omitted), with $g = D(h)$; then, $\varkappa\langle x, \pi\rangle$ requires that π_h holds with z_g^j at u_g+1. The inductive hypothesis implies that $u_{g-1} + 1 \leq i + d_{g-1} \leq u_g$, and $\varkappa\langle z, \mathsf{Pn}\rangle$ and the definition of δ_g guarantee that $u_g < i+d_g$. Correspondingly, add $\theta \triangleq i + d_g - u_g - 1$ stuttering steps at position u_g in w. This "shifts" the previous position u_g+1 to the new position $i+d_g$; hence $w, i+d_g \models \pi_h$ and $i+d_g \leq d_{g+1}$ because we added only stuttering steps. Also, $w \models \phi'$ is still the case, because Lemma 7 guarantees that the removal or addition of stuttering steps to w do not affect the satisfiability of ϕ'. Finally, observe that we introduced no more than m nss over every subword of w of length $\mathbb{V}$, and $m \leq M \leq D$ because of the pigeonhole principle, hence the variability of propositions $\mathcal{Q}$ in w is bounded by $\mathbb{V}/D$.

In all, induction proves that the finally modified w is such that $w \models \eta$ and $w \in \mathbf{var}(\mathcal{Q}, \mathbb{V}/D)$. ■

Example 12. Consider the running example and transform Ω (Example 4) into Ω' according to the above construction. Table I shows a partial model for Ω', where all propositions not appearing at some position are assumed to be false there, nss are in bold and underlined, while a hat marks successors of nss; also, $\overline{x}$ denotes $\neg x$ for every proposition x. It should be clear that the model can be transformed into one satisfying Ω, such as the one in Example 4. For example, the metric requirement that e occur once at $1460+1-40 = 1421$ is accommodated by removing all the stuttering steps at position 8 and by adding $1421 - 8 = 1413$ additional stuttering steps at position 2. $\diamondsuit$

VI. Future work

Future work will investigate possible generalizations and consider implementations. We will consider extensions of the results of the present paper to: (a) *subword* stuttering [4], where a subword is repeated multiple times, such as in the word $abc\, abc\, abc\, \cdots$; (b) Büchi automata and the classical linear-time model-checking problem. We also plan to: (a) implement a translator from LTL to formulas equi-satisfiable over words with bounded variability and combine it with off-the-shelf LTL satisfiability checking tools; (b) formalize systems characterized by time granularity heterogeneity, in order to determine how often the assumption of "sparse" events is compatible with accurate models thereof.

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References

[1] L. Lamport, "What good is temporal logic?" in *Proceedings of the IFIP 9th World Computer Congress on Information Processing (IFIP'83)*. North Holland/IFIP, 1983, pp. 657–668.

[2] D. Peled and T. Wilke, "Stutter-invariant temporal properties are expressible without the next-time operator," *Information Processing Letters*, vol. 63, no. 5, pp. 243–246, 1997.

[3] K. Etessami, "A note on a question of Peled and Wilke regarding stutter-invariant LTL," *Information Processing Letters*, vol. 75, no. 6, pp. 261–263, 2000.

[4] A. Kučera and J. Strejček, "The stuttering principle revisited," *Acta Informatica*, vol. 41, no. 7/8, pp. 415–434, 2005.

[5] A. P. Sistla and E. M. Clarke, "The complexity of propositional linear temporal logics," *Journal of the ACM*, vol. 32, no. 3, pp. 733–749, 1985.

[6] S. Demri and P. Schnoebelen, "The complexity of propositional linear temporal logics in simple cases," *Information and Computation*, vol. 174, no. 1, pp. 84–103, 2002.

[7] Y. Hirshfeld and A. M. Rabinovich, "Logics for real time: Decidability and complexity," *Fundamenta Informaticae*, vol. 62, no. 1, pp. 1–28, 2004.

[8] ——, "Timer formulas and decidable metric temporal logic," *Information and Computation*, vol. 198, no. 2, pp. 148–178, 2005.

[9] P. Bouyer, N. Markey, J. Ouaknine, and J. Worrell, "On expressiveness and complexity in real-time model checking," in *Proceedings of the 35th International Colloquium on Automata, Languages and Programming (ICALP'08)*, ser. LNCS, vol. 5126. Springer, 2008, pp. 124–135.

[10] C. A. Furia, D. Mandrioli, A. Morzenti, and M. Rossi, "Modeling time in computing: a taxonomy and a comparative survey," *ACM Computing Surveys*, vol. 42, no. 2, pp. 1–59, February 2010, article 6. Also available as http://arxiv.org/abs/0807.4132.

[11] C. A. Furia and P. Spoletini, "On relaxing metric information in linear temporal logic," http://arxiv.org/abs/0906.4711, 2011.

[12] D. M. Gabbay, I. Hodkinson, and M. Reynolds, *Temporal Logic (vol. 1): mathematical foundations and computational aspects*, ser. Oxford Logic Guides. Oxford University Press, 1994, vol. 28.

[13] E. A. Emerson, "Temporal and modal logic," in *Handbook of Theoretical Computer Science*. Elsevier Science, 1990, vol. B, pp. 996–1072.

[14] Y. Kesten, Z. Manna, and A. Pnueli, "Temporal verification of simulation and refinement," in *A Decade of Concurrency, Reflections and Perspectives, REX School/Symposium*, ser. LNCS, vol. 803. Springer, 1994.

[15] K. Etessami, "Stutter-invariant languages, omega-automata, and temporal logic," in *Computer Aided Verification, 11th International Conference, CAV '99, Trento, Italy, July 6-10, 1999, Proceedings*, ser. LNCS, vol. 1633. Springer, 1999, pp. 236–248.

[16] A. M. Rabinovich, "Expressive completeness of temporal logic of action," in *Mathematical Foundations of Computer Science 1998, 23rd International Symposium, MFCS'98, Brno, Czech Republic, August 24-28, 1998, Proceedings*, ser. LNCS, vol. 1450. Springer, 1998, pp. 229–238.

[17] C. Dax, F. Klaedtke, and S. Leue, "Specification languages for stutter-invariant regular properties," in *Automated Technology for Verification and Analysis, 7th International Symposium, ATVA 2009, Macao, China, October 14-16, 2009. Proceedings*, ser. LNCS, vol. 5799. Springer, 2009, pp. 244–254.

[18] N. Markey, "Past is for free: on the complexity of verifying linear temporal properties with past," *Acta Informatica*, vol. 40, no. 6-7, pp. 431–458, 2004.

[19] M. Bauland, T. Schneider, H. Schnoor, I. Schnoor, and H. Vollmer, "The complexity of generalized satisfiability for linear temporal logic," *Logical Methods in Computer Science*, vol. 5, no. 1, 2008.

$\underline{\mathbf{1}}$	$\widehat{2}$	3	4	5	$\underline{\mathbf{6}}$	$\underline{\widehat{\mathbf{7}}}$	$\widehat{8}$	9	$\underline{\mathbf{10}}$	$\widehat{11}$	$\underline{\mathbf{12}}$	$\underline{\widehat{\mathbf{13}}}$	$\widehat{14}$
s	$\overline{s}$	$\overline{s}$	$\overline{s}$	$\overline{s}$	$\overline{s}$	s	$\overline{s}$	$\overline{s}$	$\overline{s}$	s	s	$\overline{s}$	s
y_1	y_2 z_1^1	y_2 z_1^1	y_2 z_1^1	y_2 z_1^1	y_2 z_2^1	y_3 z_3^1 z_1^2	y_4 z_3^1 z_2^2 z_1^3	y_4 z_3^1 z_2^2 z_1^3	y_4 z_3^1 z_2^2 z_1^3	y_5 z_3^1 z_2^2 z_1^3 z_1^4	y_5 z_3^1 z_2^2 z_1^3 z_1^4	y_6 z_3^1 z_2^2 z_2^3 z_1^4 z_1^5	y_1 z_4^1 z_3^2 z_2^3 z_2^4 z_1^5 z_1^6
$x_2 \wedge x_5$						x_4	x_3	x_3	$x_1 \wedge x_3$	x_1	x_1	x_1	x_1
$\overline{v}$	v	v	v	v	v	v	v	v	v	v	v	v	$\overline{v}$
q	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	$\overline{q}$	q
$\overline{e}$	$\overline{e}$	$\overline{e}$	$\overline{e}$	$\overline{e}$	$\overline{e}$	$\overline{e}$	e	e	e	$\overline{e}$	$\overline{e}$	$\overline{e}$	$\overline{e}$
u	$\overline{u}$	$\overline{u}$	$\overline{u}$	$\overline{u}$	$\overline{u}$	$\overline{u}$	$\overline{u}$	$\overline{u}$	$\overline{u}$	u	u	u	u

Table I
A PARTIAL MODEL OF Ω' IN EXAMPLE 12.

[20] E. Corsetti, E. Crivelli, D. Mandrioli, A. Morzenti, A. Montanari, P. San Pietro, and E. Ratto, "Dealing with different time scales in formal specifications," in *In Proceedings of the 6th International Workshop on Software Specification and Design*, 1991, pp. 92–101.

[21] U. Dal Lago and A. Montanari, "Calendars, time granularities, and automata," in *Advances in Spatial and Temporal Databases, 7th International Symposium, SSTD 2001, Redondo Beach, CA, USA, July 12-15, 2001, Proceedings*, ser. LNCS, vol. 2121, 2001, pp. 279–298.

[22] C. Combi, M. Franceschet, and A. Peron, "Representing and reasoning about temporal granularities," *Journal of Logic and Computation*, vol. 14, no. 1, pp. 51–77, 2004.

[23] A. Burns and I. J. Hayes, "A timeband framework for modelling real-time systems," *Real-Time Systems*, Accepted for publication, 2010.

[24] A. Pnueli and A. Zaks, "On the merits of temporal testers," in *25 Years of Model Checking*, ser. LNCS, vol. 5000. Springer, 2008, pp. 172–195.

[25] T. Wilke, "Specifying timed state sequences in powerful decidable logics and timed automata," in *Proceedings of the 3rd International Symposium on Formal Techniques in Real-Time and Fault-Tolerant Systems (FTRTFT'94)*, ser. LNCS, vol. 863. Springer-Verlag, 1994, pp. 694–715.

[26] M. Fränzle, "Decidability of duration calculi on restricted model classes," Institut für Informatik und Praktische Mathematik, Christian-Albrechts Universität Kiel, Tech. Rep. Kiel MF 21/1, July 1996.

[27] C. A. Furia and M. Rossi, "MTL with bounded variability: Decidability and complexity," in *Proceedings of the 6th International Conference on Formal Modelling and Analysis of Timed Systems (FORMATS'08)*, ser. LNCS, vol. 5215. Springer-Verlag, September 2008, pp. 109–123.

[28] C. Colombo, G. J. Pace, and G. Schneider, "Safe runtime verification of real-time properties," in *Formal Modeling and Analysis of Timed Systems, 7th International Conference*, ser. LNCS, vol. 5813, 2009, pp. 103–117.

[29] R. Alur and T. A. Henzinger, "Logics and models of real time: A survey," in *Proceedings of Real-Time: Theory in Practice*, ser. LNCS, vol. 600. Springer-Verlag, 1992, pp. 74–106.

[30] F. Laroussinie, A. Meyer, and E. Petonnet, "Counting LTL," in *Proceedings of the 17th International Symposium on Temporal Representation and Reasoning (TIME'10)*. IEEE Computer Society Press, 2010.

[31] A. Bouajjani, R. Echahed, and P. Habermehl, "On the verification problem of nonregular properties for nonregular processes," in *Proceedings, Tenth Annual IEEE Symposium on Logic in Computer Science, San Diego, California, USA, 26-29 June 1995*. IEEE Computer Society, 1995, pp. 123–133.